

SIMONS COLLABORATION ON HOMOLOGICAL MIRROR SYMMETRY

MIRROR SYMMETRY AND STABILITY

CONFERENCE PROGRAM

MONDAY, MARCH 18

9:00 - 9:30 am Morning Refreshments

9:30-10:30 AM EMANUELE MACRÌ, NORTHEASTERN UNIVERSITY & INSTITUTE DES HAUTES ÉTUDES SCIENTIFIQUES (IHES)

Title: “Derived categories of cubic fourfolds and non-commutative K3 surfaces”

Abstract: The derived category of coherent sheaves on a cubic fourfold has a subcategory, which can be thought as the derived category of a non-commutative K3 surface. This subcategory was studied recently in the work of Kuznetsov and Addington-Thomas, among others. In this talk, I will present joint work with Bayer, Lahoz, Nuer, Perry, Stellari, on how to construct Bridgeland stability conditions on this subcategory. This proves a conjecture by Huybrechts, and it allows to start developing the moduli theory of semistable objects in these categories, in an analogue way as for the classical Mukai theory for (commutative) K3 surfaces. I will also discuss a few applications of these results.

10:30 - 11:00 am Coffee Break

11:00-12:00 AM LAURA PERTUSI, THE UNIVERSITY OF EDINBURG

Title: “Rational curves of low degree on cubic fourfolds and stability conditions.”

Abstract: A famous result of Beauville and Donagi states that the Fano variety of lines on a cubic fourfold is a smooth projective irreducible holomorphic symplectic (IHS) variety of dimension four, equivalent by deformation to the Hilbert square on a K3 surface. More recently, Lehn, Lehn, Sorger and van Straten constructed an IHS eightfold of K3 type from twisted cubic curves on a cubic fourfold Y non containing a plane.

In this talk, I will give an interpretation of the Fano variety of lines and of the LLSvS eightfold as moduli spaces of Bridgeland stable objects in the Kuznetsov component of Y . As a consequence, we reprove the categorical version of Torelli Theorem for cubic fourfolds, we obtain the identification of the period point of the LLSvS eightfold with that of the Fano variety, and we discuss the derived Torelli Theorem for cubic fourfolds. This is a joint work with Chunyi Li and Xiaolei Zhao.

12:00 - 2:00 pm Catered lunch

2:00-3:00 PM XIAOLEI ZHAO, UC SANTA BARBARA

Title: “Elliptic quintics on cubic fourfolds, O’Grady 10, and Lagrangian fibration”

Abstract: We study elliptic quintics on a generic cubic fourfold, and show that their moduli space is naturally related to some hyper-Kähler 10-fold, deformation equivalent to O’Grady’s example. Moreover, we study the existence of Lagrangian fibration on this moduli space and its connection to the intermediate Jacobians of cubic threefolds. This is based on a work in progress with Chunyi Li and Laura Pertusi.

3:00 - 3:30 pm Coffee Break

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3:30–4:30 PM JACOPO STOPPA, SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI (SISSA)

Title: “Variations of BPS structure and enumerative geometry”

Abstract: A “variation of BPS structure” is a nice name for the kind of infinite dimensional bundle with connection one can construct, at least formally, starting from the Donaldson-Thomas type invariants of a Calabi-Yau threefold. In the first part of the talk I will offer an introduction to this circle of ideas. Then I will focus on the concrete example of what happens in this construction when we start with the DT invariants counting 1-dimensional torsion sheaves, or more generally sheaf-theoretic Gopakumar-Vafa invariants. The answer is closely related to the Gromov-Witten partition function.

6:00pm Banquet at Mu Lan restaurant, 228 Broadway, Cambridge, MA 02139

TUESDAY, MARCH 19

9:00 - 9:30 am Morning Refreshments

9:30–10:30 AM CHUNYI LI, THE UNIVERSITY OF WARWICK

Title: “Stronger Bogomolov-Gieseker type Inequality and stability condition”

Abstract: The classical Bogomolov inequality gives a bound for the second Chern character of slope stable sheaves on smooth projective varieties. The inequality is known to be sharp for some varieties (e.g. Abelian varieties), as well as non-sharp for some others (e.g. the projective plane). Besides Fano and K3 surfaces, it is always difficult to get stronger Bogomolov type inequalities for other surfaces and higher dimensional varieties. I will talk about the method to set up such inequalities via the Bridgeland stability condition.

The stronger Bogomolov type inequality has several implications. One upshot will be the existence of stability condition on smooth quintic threefolds. They are the first examples of Calabi-Yau threefolds with trivial fundamental group known to have stability conditions.

10:30 - 11:00 am Coffee Break

11:00–12:00 PM NAOKI KOSEKI, THE UNIVERSITY OF TOKYO

Title: “Stability conditions on threefolds with nef tangent bundles”

Abstract: The construction of Bridgeland stability conditions on a given threefold is an open problem in general. The problem is reduced to proving the so-called Bogomolov-Gieseker (BG) type inequality conjecture proposed by Bayer-Macri-Toda. In this talk, I will explain how to prove the BG type inequality conjecture for threefolds in the title.

12:00 - 2:00 pm Catered lunch

2:00–3:00 PM GENKY OUCHI, RIKEN INTERDISCIPLINARY THEORETICAL AND MATHEMATICAL

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Title: “Symplectic automorphism groups of cubic fourfold and K3 categories”

Abstract: Gaberdiel, Hohenegger and Volpato (GHV) characterized automorphism groups of K3 sigma models in terms of Mukai lattice and Leech lattice. Huybrechts gave a geometric interpretation of GHV Theorem in terms of derived categories of K3 surfaces and Bridgeland stability conditions on them. In this talk, I would like to characterize symplectic automorphism groups of cubic fourfolds as automorphism groups of certain K3 sigma models using Bridgeland stability conditions on Kuznetsov’s K3 categories due to Bayer, Lahoz, Macri and Stellari.

3:00–4:00 PM DYLAN ALLEGRETTI, THE UNIVERSITY OF SHEFFIELD

Title: “Relating stability conditions and cluster varieties

Abstract: Associated to a quiver with potential are two interesting spaces. The first is a complex manifold parametrizing Bridgeland stability conditions on a triangulated category, and the second is a cluster variety with a natural Poisson structure. The structure of each space is controlled by the combinatorics of quiver mutations, but the combinatorics is used quite differently in the two cases. Whereas the space of stability conditions has a wall-and-chamber decomposition, the cluster variety is defined as a union of algebraic tori glued by birational maps. In this talk, I will describe the relationship between these two spaces for quivers of Dynkin type A. Using ideas from the theory of ordinary differential equations, I will construct a local biholomorphism from the space of stability conditions to the cluster variety and explain how this map relates the structures of the two spaces.

4:00 - 4:30 pm Coffee Break

4:30–5:30 PM ALEX TAKEDA, UC BERKELEY

Title: “Using relative constructions to understand stability conditions on Fukaya categories of surfaces”

Abstract: In this talk I will present the techniques and results from arXiv:1811.10592, where a new notion of a relative stability condition is presented. This is defined in analogy with compactly supported chains, and using this tool we are able to prove that the stability conditions defined by Haiden, Katzarkov and Kontsevich using quadratic differentials cover the entire stability space of that surface's Fukaya category, in the so-called "fully stopped case". I will also talk about the space of such relative stability conditions and prove some of its nice properties, some of which generalize the properties held by spaces of ordinary Bridgeland stability conditions. The definition of these relative versions has only been worked out in the setting of these categories; time allowing I will discuss some expectations and hopes for extending it to a broader context.

WEDNESDAY, MARCH 20

9:00 - 9:30 am Morning Refreshments

9:30–10:30 AM TRISTAN COLLINS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY (MIT)

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Title: “Stability and Nonlinear PDE in mirror symmetry”

Abstract: A longstanding problem in mirror symmetry has been to understand the relationship between the existence of solutions to certain geometric nonlinear PDES (the special Lagrangian equation, and the deformed Hermitian-Yang-Mills equation) and algebraic notions of stability, mainly in the sense of Bridgeland. I will discuss progress in this direction through ideas originating in infinite dimensional GIT. This is joint work with S.-T. Yau.

10:30 - 11:00 am Coffee Break

11:00-12:00 PM JASON LO, CALIFORNIA STATE UNIVERSITY, NORTHRIDGE (CSUN)

Title: “Limit stability conditions on elliptic threefolds”

Abstract: On the derived category of coherent sheaves on a projective variety, constructions of Bridgeland stability conditions involve an ample class as a parameter. When we let this ample class approach infinity along a ray, we obtain a notion of stability that is often called the “large volume limit”. On certain elliptic threefolds, it is possible to take a different limit in the ample class and obtain a different notion of stability. In this talk, I will discuss the connections between these two types of limits, and possible implications on moduli spaces such as those of stable pairs.