A function of two variables \( f(x, y) \) is a rule which assigns to two numbers \( x, y \) a third number \( f(x, y) \). For example, the function \( f(x, y) = x^2y + 2x \) assigns to \((3, 2)\) the number \( 3^2 \cdot 2 + 6 = 24 \). The domain \( D \) of a function is set of points where \( f \) is defined, the range is \( \{ f(x, y) \mid (x, y) \in D \} \). The graph of \( f(x, y) \) is the surface \( \{(x, y, f(x, y)) \mid (x, y) \in D \} \) in space. Graphs allow to visualize functions.

1. The graph of \( f(x, y) = \sqrt{1 - (x^2 + y^2)} \) on the domain \( D = \{ x^2 + y^2 < 1 \} \) is a half sphere. The range is the interval \([0, 1]\).

The set \( f(x, y) = c = \text{const} \) is called a contour curve or level curve of \( f \). For example, for \( f(x, y) = 4x^2 + 3y^2 \), the level curves \( f = c \) are ellipses if \( c > 0 \). The collection of all contour curves \( \{ f(x, y) = c \} \) is called the contour map of \( f \).

2. For \( f(x, y) = x^2 - y^2 \), the set \( x^2 - y^2 = 0 \) is the union of the lines \( x = y \) and \( x = -y \). The curve \( x^2 - y^2 = 1 \) is made of two hyperbola with with their "noses" at the point \((-1, 0)\) and \((1, 0)\). The curve \( x^2 - y^2 = -1 \) consists of two hyperbola with their noses at \((0, 1)\) and \((0, -1)\).

3. For \( f(x, y) = (x^2 - y^2)e^{-x^2-y^2} \), we can not find explicit expressions for the contour curves \((x^2 - y^2)e^{-x^2-y^2} = c\). but we can draw the curves with the computer:

A function of three variables \( g(x, y, z) \) assigns to three variables \( x, y, z \) a real number \( g(x, y, z) \). We can visualize it by contour surfaces \( g(x, y, z) = c \), where \( c \) is constant. It is helpful to look at the traces, the intersections of the surfaces with the coordinate planes \( x = 0, y = 0 \) or \( z = 0 \).

4. For \( g(x, y, z) = z - f(x, y) \), the level surface \( g = 0 \) which is the graph \( z = f(x, y) \) of a function of two variables. For example, for \( g(x, y, z) = z - x^2 - y^2 = 0 \), we have the graph \( z = x^2 + y^2 \) of the function \( f(x, y) = x^2 + y^2 \) which is a paraboloid. Most surfaces \( g(x, y, z) = c \) are not graphs.
If \( f(x, y, z) \) is a polynomial and \( f(x, x, x) \) is quadratic in \( x \), then \( \{ f = c \} \) is a **quadric**.

\[
x^2 + y^2 + z^2 = 1
\]

**Sphere**

\[
x^2 + y^2 - c = z
\]

**Paraboloid**

\[
ax + by + cz = d
\]

**Plane**

\[
x^2 + y^2 - z^2 = 1
\]

**One sheeted Hyperboloid**

\[
x^2 + y^2 = r^2
\]

**Cylinder**

\[
x^2 + y^2 - z^2 = -1
\]

**Two sheeted Hyperboloid**

\[
x^2/a^2 + y^2/b^2 + z^2/c^2 = 1
\]

**Ellipsoid**

\[
x^2 - y^2 + z = 1
\]

**Hyperbolic paraboloid**

\[
x^2/a^2 + y^2/b^2 - z^2/c^2 = 1
\]

**Elliptic hyperboloid**