14. $x = \frac{18}{5}$, $y = \frac{2}{5}$, optimal value of $z$ is $\frac{58}{5}$.

16. $\frac{3}{2}$ tons of regular steel and $\frac{3}{2}$ tons of special steel; maximum profit is $430$.

18. Invest $4000$ in bond A and $2000$ in bond B; maximum return is $520$.

20. Use 2 minutes of advertising and 28 minutes of programming; maximum number of viewer-minutes is 1,340,000.

22. Use 4 units of A and 3 units of B; maximum amount of protein is 34 units.

24. (b).

26. Maximize $z = 2x_1 - 3x_2 - 2x_3$

subject to

\begin{align*}
2x_1 + x_2 + 2x_3 & \leq 12 \\
x_1 + x_2 - 3x_3 & \leq 8 \\
x_1 & \geq 0, x_2 \geq 0, x_3 \geq 0
\end{align*}

28. Maximize $z = 2x + 8y$

subject to

\begin{align*}
2x + 3y + u & = 18 \\
3x - 2y + v & = 6 \\
x & \geq 0, y & \geq 0, u & \geq 0, v & \geq 0
\end{align*}

Section 11.2, p. 589

2.

<table>
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<tr>
<th>$x_1$</th>
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<th>$x_3$</th>
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6. $x = 0, y = \frac{8}{7}$, optimal $z$ value is $\frac{30}{7}$.

8. No finite optimal solution.

10. $x_1 = 0, x_2 = \frac{5}{2}, x_3 = 0$, optimal $z$ value is $z = 10$.

12. $\frac{3}{2}$ tons of regular steel and $\frac{5}{2}$ tons of special steel; maximum profit is $\$430$.

14. 4 units of A and 3 units of B; maximum amount of protein is 34 units.

T.1. We must show that if $x$ and $y$ are any two feasible solutions, then for any $0 \leq r \leq 1$, the vector $rx + (1 - r)y$ is also a feasible solution. First, since $r > 0$ and $(1 - r) > 0$, and $Ax \leq b, Ay \leq b$,

$$A[rx + (1 - r)y] = rAx + (1 - r)Ay \leq rb + (1 - r)b = b.$$ 

Also, since $x \geq 0, y \geq 0$,

$$rx + (1 - r)y \geq r \cdot 0 + (1 - r) \cdot 0 = 0.$$ 

Thus $rx + (1 - r)y$ is a feasible solution.

T.2. Suppose that in a certain step of the simplex method, the minimum positive $\theta$-ratio is not chosen. After a reindexing of the variables, if necessary, we may assume that all the nonbasic variables occur first followed by all the basic variables. That is, we may assume that we start with the situation given by Tableau 2 which precedes Equation (16). Let us further assume that $x_1$ is the entering variable and $x_{n+1}$ the departing variable and that the $\theta$-ratio $b_2/a_{21}$ associated with the second row is positive and smaller than $b_1/a_{11}$ associated with the first row — the row of the incorrectly chosen departing variable $x_{n+1}$. Thus

$$0 < \frac{b_2}{a_{21}} < \frac{b_1}{a_{11}}$$

$$a_{11}b_2 < a_{21}b_1$$

$$a_{11}b_2 - a_{21}b_1 < 0.$$ 

The new set of nonbasic variables is \{ $x_2, \ldots, x_n, x_{n+1}$ \}. Set these nonbasic variables equal to zero and solve the first equation for $x_1$:

$$a_{11}x_1 + a_{12} \cdot 0 + \cdots + a_{1n} \cdot 0 + 0 = b_1$$

$$x_1 = \frac{b_1}{a_{11}}.$$ 

Next substitute this value for $x_1$ into the second equation and solve for the basic variable $x_{n+2}$:

$$a_{21} \left( \frac{b_1}{a_{11}} \right) + a_{22} \cdot 0 + \cdots + a_{2n} \cdot 0 + x_{n+2} = b_2,$$

$$x_{n+2} = b_2 - \frac{a_{21}b_1}{a_{11}} = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}} = \text{neg} \rightarrow \text{pos} = \text{neg},$$

a contradiction to the fact that the coordinate $x_{n+2}$ of any feasible solution must be $\geq 0$. 

Section 11.2
Section 11.3, p. 598

2. Maximize $z' = 5w_1 + 6w_2$
subject to

$\begin{align*}
& w_1 + 2w_2 \geq 10 \\
& 3w_1 - 4w_2 \geq 12 \\
& 4w_1 - 5w_2 \geq 15 \\
& w_1 \geq 0, w_2 \geq 0
\end{align*}$

4. Maximize $z' = 9w_1 + 12w_2$
subject to

$\begin{align*}
& 3w_1 + 5w_2 \leq 14 \\
& 5w_1 + 2w_2 \leq 12 \\
& -4w_1 + 7w_2 \leq 18 \\
& w_1 \geq 0, w_2 \geq 0
\end{align*}$

6. $w_1 = \frac{2}{3}, w_2 = 0$, optimal value is 4.

8. $w_1 = \frac{1}{10}, w_2 = \frac{7}{10}$, optimal value is $\frac{69}{10}$.

10. Use 6 oz. of dates and no nuts or raisins. Total cost is 90 cents.

Section 11.4, p. 612

2. 

<table>
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<th></th>
<th>stone</th>
<th>scissors</th>
<th>paper</th>
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