EXERCISE 2.3.21: MULTIPLYING BY THE CONJUGATE

SEBASTIEN VASEY

Exercise

Compute the following limit (if it exists):

$$\lim_{h \to 0} \frac{\sqrt{9 + h} - 3}{h}$$

Solution

Plugging in $h = 0$ leads an indeterminate expression of the form $\frac{0}{0}$, so some simplification is needed. We use a simple trick worth remembering: we multiply by one, writing one as something which is convenient to simplify the expression.

$$\lim_{h \to 0} \frac{\sqrt{9 + h} - 3}{h} = \lim_{h \to 0} \frac{\sqrt{9 + h} - 3}{h} \cdot 1$$

$$= \lim_{h \to 0} \frac{\sqrt{9 + h} - 3}{h} \cdot \frac{\sqrt{9 + h} + 3}{\sqrt{9 + h} + 3}$$

$$= \lim_{h \to 0} \frac{9 + h - 9}{h(\sqrt{9 + h} + 3)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{9 + h} + 3)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{9 + h} + 3}$$

$$= \frac{1}{\sqrt{9 + 0} + 3}$$

$$= \frac{1}{6}$$

$\sqrt{9 + h} + 3$ is called the conjugate of $\sqrt{9 + h} - 3$. Multiplying by the conjugate is sometimes used to get rid of square roots, as seen in this example.