(Math 360) Homework 4:

Due February 12, 2009

All numbered exercises are from Rudin's Principles of Mathematical Analysis.

Exercise 1: Chapter 2, Exercise 21.

Exercise 2: Chapter 2, Exercise 23.

Exercise 3: Chapter 2, Exercise 24.

Exercise 4: Chapter 2, Exercise 25.

Exercise 5: Let (X, d) be a metric space. Show that every closed set $C \subseteq X$ is the intersection of countably many open sets. (Hint: Consider $U_q = \bigcup_{x \in C} B(x, q)$ where $q \in \mathbb{Q}$)

Exercise 6: Let $x_n \to x$ be a convergent sequence in a metric space. Let $A = \mathrm{dom}(\{x_n\}) \cup \{x\} = \{x_1, x_2, \dots\} \cup \{x\}$. Show that A is compact.

Exercise 7: Let (X, d) be a metric space and for all $n \in \mathbb{N}$ let $\emptyset \neq U_n \subseteq X$ be such that

- U_n is open in X
- The closure of $U_{n+1} = \overline{U_{n+1}} \subseteq U_n$ for all $n \in \mathbb{N}$
- $\overline{U_n}$ is compact for all $n \in \mathbb{N}$

Show that $\bigcap_{n=0}^{\infty} U_n \neq \emptyset$

<u>Exercise 8:</u> Let (X, d) be a metric space. Show that if $K_1, \ldots, K_n$ is a finite collection of compact subsets of (X, d) then $\bigcup_{i=1}^{n} K_i$ is also a compact subset of (X, d).