(Math 360) Homework 10:

Due April 9, 2009

All numbered exercises are from Rudin's Principles of Mathematical Analysis.

Exercise 1: Chapter 6, Exercise 4.

Exercise 2: Chapter 6, Exercise 6.

Exercise 3: Chapter 6, Exercise 11.

Exercise 4: Chapter 6, Exercise 12.

Exercise 5: Chapter 6, Exercise 16.

Suppose $A \subseteq [a, b] \subseteq R$. We define the *characteristic function of* A to be the function ξ_A such that $\xi_A(x) = 1$ if $x \in A$ and $\xi_A(x) = 0$ if $x \notin A$.

We further say A has *length* if $\int_a^b \xi_A(x)dx$ is defined. We then say the *length of* A is the value of the integral and we denote it by $\mu(A)$.

We say that A has *measure zero* if for every $\epsilon > 0$ there are countably many intervals $\langle S_i : i \in \mathbb{N}\rangle$ such that

- $\sum_{i=1}^{\infty} \mu(S_i) < \epsilon$
- $A \subseteq \bigcup_{i=1}^{\infty} S_i$

Exercise 7: (a) Prove that the set of rations between 0 and 1, $\mathbb{Q} \cap [0, 1]$, does not have length.

(b) Prove that $\mathbb{Q} \cap [0,1]$ has measure zero (Hint: the interval $[a,a]$ is the single point $\{a\}$)

Exercise 8: Suppose $A \subseteq [a,b]$ has measure zero. Prove $[a,b] - A = \{x \in [a,b] : x \notin A\}$ does not have measure zero.

Exercise 9: (a) Suppose $f, g \in \mathcal{R}$ with the domain of f, g equal to $[a,b]$. Further suppose that $S = \{x \in [a,b] : f(x) \neq g(x)\}$ has measure 0. Then show that $\int_a^b f(x)dx = \int_a^b g(x)dx$.

(b) Suppose $f, g \in \mathcal{R}$ are bounded and $\int_a^b |f(x) - g(x)|dx = 0$ then show $S = \{x \in [a,b] : f(x) \neq g(x)\}$ has measure 0.