(Math 360) Multiple Choice Final:

April 20, 2009

Write all answers in the spaces provided below! No notes or calculators allowed. There is no penalty for guessing and there is no partial credit. Good Luck!

Name ____

1. ________	8. ________	15. ________
2. ________	9. ________	16. ________
3. ________	10. ________	17. ________
4. ________	11. ________	18. ________
5. ________	12. ________	19. ________
6. ________	13. ________	20. ________
7. ________	14. ________	

Score: ________ (100 points possible)

(1) Which properties holds of the set of all complex numbers z such that $|z| < 1$?

(I) The set is closed in $\mathbb{R}^2$

(II) The set is open in $\mathbb{R}^2$

(III) The set is bounded in $\mathbb{R}^2$

(a) (I)

(b) (II)

(c) (I) and (III)

(d) (II) and (III)

(2) Which of the following is NOT compact?

(a) Every finite set

(b) $\{\mathbf{x} \in \mathbb{R}^k : |x| \leq 1\}$

(c) $[0, \infty)$

(3) Suppose $\{U_i : i \in \mathbb{N}\}$ are open subsets of a metric space such that the intersection of every finite subcollection is non-empty. Must $\bigcap_{i \in \mathbb{N}} U_i$ be non-empty?

(Y) Yes

(N) No

(4) Must any two disjoint sets be separated?

(Y) Yes

(N) No

(5) If $x \in (-1, 1)$ what does $\sum_{i=0}^{\infty} x^{2i}$ equal?

(a) $\frac{1}{1-x^2}$

(b) $\frac{x}{1-x^2}$

(c) $\frac{1}{1+x}$

(c) $\frac{x}{1-x}$

(6) Suppose $f : (X, d_X) \to (Y, d_Y)$ is a map such that for every closed set $U \subseteq Y$, $f^{-1}(U)$ is closed. What property can we say for sure that f has?

(a) f is an isometry

(b) f is a continuous

(c) f is a uniformly continuous

(d) None of the above

(7) Do there exists metric spaces X and Y with X compact and a continuous mapping $f : X \to Y$ such that $f(X)$ is NOT compact?

(Y) Yes

(N) No

(8) Must every continuous one-to-one map with a continuous inverse be an isometry?

(Y) Yes

(N) No

(9) Is $\mathbb{Q} \cap [0,1]$ a compact set

(Y) Yes

(N) No

(10) If $f(x), g(x) : \mathbb{R} \to \mathbb{R}$ are uniformly continuous functions must $f(g(x))$ be uniformly continuous?

(Y) Yes

(N) No

(11) Suppose P, P^* are partitions of $[a,b]$ such that P^* is a refinement of P. If $f : [a,b] \to \mathbb{R}$ is a function, which of the following must hold?

(I) $L(P,f) \leq U(P^*,f)$

(II) $L(P^*,f) \leq U(P,f)$

(III) $L(P^*,f) \geq L(P,f)$

(IV) $U(P^*,f) \geq U(P,f)$

(a) (I)

(b) (I) and (II)

(c) (I), (II) and (III)

(d) (I), (II) and (IV)

(12) Suppose $F(x) = \int_a^x f(t)dt$ and f is continuous at x_0. Must F be differentiable at x_0?

(Y) Yes

(N) No

(13) Suppose $\{f_n\}$ is a sequence of integrable functions on $[a, b]$ such that $(\forall x \in [a, b]) \lim_{n\to\infty} f_n(x) = f(x)$. Must

$$\int_a^b f\ dx = \lim_{n\to\infty} \int_a^b f_n\ dx?$$

(Y) Yes

(N) No

(14) Is there a continuous map from $[0, 1]$ onto $(0, 1)$ (i.e. where every element of $(0, 1)$ is in the range)?

(Y) Yes

(N) No

(15) Is there a metric d on $\mathbb{R}$ such that every function $f : (\mathbb{R}, d) \to (\mathbb{R}, d)$ is continuous?

(Y) Yes

(N) No

(16) If X is a metric space and $C \subseteq X$ is closed must there exist a continuous function $f : X \to R$ such that $\{x : f(x) = 0\} = C$?

(Y) Yes

(N) No

(17) Must a uniformly continuous and differentiable function have a bounded derivative on it's domain?

(Y) Yes

(N) No

(18) If X is disconnected and $f : X \to Y$ is continuous is $f(X)$ necessarily disconnected?

(Y) Yes

(N) No

(19) If $f : X \to Y$ is continuous and injective (i.e. one-to-one) must f^{-1} be continuous?

(Y) Yes

(N) No

(20) If $A \subseteq \mathbb{R}$ and every continuous function $f : A \to \mathbb{R}$ is bounded, must A be compact?

(Y) Yes

(N) No