(Math 360) Multiple Choice Final:

April 23, 2009

Write all answers in the spaces provided below! No notes or calculators allowed. There is no penalty for guessing and there is no partial credit. Good Luck!

Name ____

1. ________	8. ________	15. ________
2. ________	9. ________	16. ________
3. ________	10. ________	17. ________
4. ________	11. ________	18. ________
5. ________	12. ________	19. ________
6. ________	13. ________	20. ________
7. ________	14. ________	

Score: ________ (100 points possible)

(1) Which of the following are true:

(a) It is possible to place an linear order relation $\leq$ on the complex numbers $\mathbb{C}$ such that $(\mathbb{C}, +, \times, \leq)$ is not an ordered field.

(b) It is possible to place a linear order relation on the complex numbers $\mathbb{C}$ which make $(\mathbb{C}, +, \times, \leq)$ an ordered field.

(c) Neither (a) nor (b)

(d) Both (a) and (b).

(2) Which of the following is not always true for $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^k$?

(a) $|\mathbf{x}| \geq 0$

(b) $|\mathbf{x} \cdot \mathbf{y}| \leq |\mathbf{x}||\mathbf{y}|$

(c) $|\mathbf{x}| = 0$ if and only if $\mathbf{x} = 0$

(d) $|\mathbf{x} + \mathbf{y}| = |\mathbf{x}| + |\mathbf{y}|$

(3) Is there a metric space X with distinct open sets $\langle U_i : i \in \mathbb{N} \rangle$ such that $\bigcap_{i \in \mathbb{N}} U_i$ is open?

(Y) Yes

(N) No

(4) Is the following statement true? "Every bounded closed subset of a metric space is compact"

(Y) Yes

(N) No

(5) Suppose $\{s_n\}$ and $\{t_n\}$ are sequences of complex numbers such that $\lim_{n \to \infty} s_n = s$ and $\lim_{n \to \infty} t_n = t$. Must $\lim_{n \to \infty} s_n t_n = st$?

(Y) Yes

(N) No

(6) Does there exist a bounded sequence of real numbers with no convergent subsequence?

(Y) Yes

(N) No

(7) Which of the following is true?

(a) Every compact space is complete

(b) Every complete space is compact

(c) Neither (a) nor (b).

(d) Both (a) and (b).

(8) Which of the following are valid intervals of convergence for a power series?

(I) $[-1, 3)$

(II) $(-\infty, 0]$

(III) $[2, 2] \cup [3, 3]$

(IV) $(-\infty, \infty)$

(a) Only (II)

(b) Only (IV)

(c) (I) and (IV)

(d) (II) and (III)

(9) Suppose $\sum |a_i|$ diverges and $\sum a_i = 2$. Is there a rearrangement a_{i_k} of the terms such that $\sum a_{i_k} = 4$?

(Y) Yes

(N) No

(10) Does there exists metric spaces X and Y with X closed and bounded and a continuous mapping $f : X \to Y$ such that $f(X)$ is NOT "closed and bounded"?

(Y) Yes

(N) No

(11) Suppose $f : X \to Y$ is continuous and X is compact. Must f be uniformly continuous?

(Y) Yes

(N) No

(12) If $f(x), g(x) : \mathbb{R} \to \mathbb{R}$ are everywhere differentiable must $f(g(x))$ be everywhere differentiable?

(Y) Yes

(N) No

(13) Suppose $f : [a, b]$ is a function and suppose f is a local maximum. Must $f'(x)$ exist and equal 0?

(Y) Yes

(N) No

(14) Suppose $f(x), g(x)$ are real differentiable functions on (a, b) and such that

$$\lim_{x \to c} \frac{f'(x)}{g'(x)} = A$$

Must

$$\lim_{x \to c} \frac{f(x)}{g(x)} = A?$$

(Y) Yes

(N) No

(15) Suppose $f \in \mathcal{R}(\alpha)$. Must $\int_a^b f \, d\alpha = \int_a^b f(x)\alpha'(x)dx$?

(Y) Yes

(N) No

(16) Suppose $\{f_n\}$ is a sequence of continuous functions on $[a, b]$ such that $(\forall x \in [a, b]) \lim_{n \to \infty} f_n(x) = f(x)$. Must f be continuous?

(Y) Yes

(N) No

(17) If $A \subseteq \mathbb{R}$ is open and $B = \overline{A}$ =closure of A, must $interior(B) = A$?

(Y) Yes

(N) No

(18) Are there any non-constant continuous maps from $\mathbb{R}$ to $\mathbb{Q}$?

(Y) Yes

(N) No

(19) If A is connected must the closure of A be connected?

(Y) Yes

(N) No

(20) Must every continuous function $f : (0, 1) \to (0, 1)$ have a fixed point?

(Y) Yes

(N) No