

Math 104
Practice Midterm Exam 3A

Name _____

Write all answers (A, B, C, D, E, F) in the spaces provided below!

1. _____

7. _____

2. _____

8. _____

3. _____

9. _____

4. _____

10. _____

5. _____

11. _____

6. _____

12. _____

1. The testing booklet contains 12 multiple choice questions and 3 free response questions.
2. No calculators are permitted.
3. One piece of paper (8.5 in. by 11 in.) is permitted, with writing on both sides allowed.
4. There is no penalty for guessing.
5. No partial credit will be given on the multiple choice questions.

1. Find the area inside *one* leaf (i.e. one loop) of the graph $r = 4\cos(3\theta)$.

A.) $\frac{2\pi}{3}$

B.) $\frac{3\pi}{4}$

C.) $\frac{\pi}{2}$

D.) $\frac{4\pi}{3}$

E.) $\frac{7\pi}{4}$

F.) 2π

2. Determine if the series is convergent or divergent. If it is convergent find the sum.

$$\sum_{n=0}^{\infty} \frac{3^n + 4^n}{7^n}$$

A.) $\frac{49}{12}$

B.) $\frac{1}{7}$

C.) $\frac{5}{12}$

D.) $\frac{12}{7}$

E.) $\frac{25}{12}$

F.) Series is divergent

3. How many of the following series converge

$$\sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n + 2} \quad \sum_{n=1}^{\infty} \frac{2^n + n^2}{3^n + n^3} \quad \sum_{n=1}^{\infty} \frac{n^2 + 3}{(n + 4)^2} \quad \sum_{n=2}^{\infty} \frac{\ln(n)}{n^2} \quad \sum_{n=2}^{\infty} \frac{n^2}{\ln(n)}$$

- A.) none
- B.) one
- C.) two
- D.) three
- E.) four
- F.) five

4. Which statement is true of the following series

$$(Ia) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1} = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

$$(II) \sum_{n=1}^{\infty} \frac{1}{n+1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$(III) \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^n}{n!} = 1 - 2 + \frac{9}{2} - \dots$$

- A.) (I) converges, (II) and (III) diverges.
- B.) (II) converges, (I) and (III) diverges.
- C.) (III) converges, (I) and (II) diverges.
- D.) (I) and (II) converges, (III) diverges.
- E.) (I) and (III) converges, (II) diverges.
- F.) (II) and (III) converges, (I) diverges.

5. Consider the following series.

$$\sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}} \quad \sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}} \quad \sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

Which of the following is true?

- A.) Only the first series converges
- B.) Only the second series converges
- C.) Only the third series converges
- D.) Both the first and second series converge.
- E.) Both the first and third series converge.
- F.) Both the second and third series converge.

6. A curve is defined by the parametric equation $x = t - t^3$, $y = 3t^2$. Notice that the curve goes through the point $(0, 3)$ for two distinct values of t and hence has a loop. Find the length of the loop.

A.) 4

B.) $\frac{8}{9}$

C.) $\frac{4\sqrt{3}}{3}$

D.) $\frac{16\sqrt{3}}{5}$

E.) $\frac{8\sqrt{3}}{15}$

F.) 0

7. Find the equation for the line tangent to the curve defined by the parametric equation $x = 1 + \frac{1}{t}$, $y = t^3 + 3$ for $t > 0$ at the point $(x, y) = (2, 4)$

A.) $y = -3x + 10$

B.) $y = -3x + 14$

C.) $y = 3x - 2$

D.) $y = 3x - 8$

E.) $y = -x/3 + 14/3$

F.) $y = -x/3 + 10/3$

8. Consider the cartesian coordinate $(x, y) = (2\sqrt{3}, -2)$. Find the polar coordinate (r, θ) where $r > 0$, and $0 \leq \theta < 2\pi$

A.) $(r, \theta) = (2, 7\pi/6)$

B.) $(r, \theta) = (4, 7\pi/6)$

C.) $(r, \theta) = (2, \pi/6)$

D.) $(r, \theta) = (2, 5\pi/6)$

E.) $(r, \theta) = (4, 5\pi/6)$

F.) $(r, \theta) = (4, 11\pi/6)$

9. Find the limit of the sequence $\{n \tan(\frac{1}{n})\}$

A.) 2

B.) 1

C.) -1

D.) $\pi/2$

E.) 0

F.) diverges

10. Define a sequence $a_1 = 2$, $a_2 = 2^{1+\frac{1}{2}}$, $a_3 = 2^{1+\frac{1}{2}+\frac{1}{4}}$, $a_n = 2^{1+\frac{1}{2}+\dots+1/2^{n-1}}$. Find $\lim_{n \rightarrow \infty} a_n$

A.) 0

B.) 1

C.) $\sqrt{2}$

D.) 2

E.) 4

F.) $+\infty$

11. Find the area inside one leaf (i.e. one loop) of the graph $r = 4 \sin(\theta)$

A.) $\frac{\pi}{8}$

B.) $\frac{\pi}{4} \ln 3$

C.) $\frac{\pi}{2} \pi$

D.) π

E.) 2π

F.) 4π

12. Find the limit of the sequence $\{\frac{1}{2} \ln(n^2 + 1) - \ln(2n + 1)\}$

A.) -2

B.) $-\ln(2)$

C.) 0

D.) $\ln(2)$

E.) 2

F.) Sequence Diverges

Free Response 1. (A) Does the series $\sum_{n=1}^{\infty} \frac{3+\cos(n)}{2^n}$ converge or diverge? Justify your answer.

(B) Does the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ converge or diverge? Justify your answer.

Free Response 2. Find the length of the curve $x = \sin(t) + \cos(t)$, $y = \sin(t) - \cos(t)$,
 $0 \leq t \leq 2\pi$

Free Response 3. The curve $r = \frac{1}{\sin(\theta)}$ breaks $r = \sqrt{2}$ into two pieces. What is the area of each piece?