Math 104
Practice Midterm Exam 2B

Name _____

Write all answers (A, B, C, D, E, F) in the spaces provided below!

1. _____ 7. _____

2. _____ 8. _____

3. _____ 9. _____

4. _____ 10. _____

5. _____ 11. _____

6. _____ 12. _____

1. The testing booklet contains 12 multiple choice questions and 3 free response questions.
2. No calculators are permitted.
3. One piece of paper (8.5 in. by 11 in.) is permitted, with writing on both sides allowed.
4. There is no penalty for guessing.
5. No partial credit will be given on the multiple choice questions.

1. Evaluate the integral.

$$\int_4^\infty \frac{1}{x^2+16}\,dx$$

A.) $\frac{\sqrt{3}}{2}$ B.) $\frac{\sqrt{3}}{2}-\frac{1}{4}$ C.) $\frac{\sqrt{3}}{2}+\frac{1}{4}$ D.) $\frac{\pi}{4}$ E.) $\frac{\pi}{16}$
F.) The integral diverges

2. Find the surface area generated by revolving the curve $y = \sqrt{1 - x^2}$ with $0 \leq x \leq 1/2$ about the x-axis.

A.) 2π

B.) π

C.) $\frac{4}{3}\pi$

D.) $\frac{\sqrt{3}}{4}\pi$

E.) 1

F.) $\frac{4}{3}$

3. Evaluate the integral if possible

$$\int_0^1 x \ln x \, dx$$

A.) -1 B.) $-1/2$ C.) $-1/4$ D.) 0 E.) 1
F.) The integral diverges

4. Evaluate the integral

$$\int_1^2 \frac{\ln x}{x^2}\, dx$$

A.) $\frac{1}{2} - \frac{1}{2}\ln 2$ B.) $-\frac{1}{2}\ln 2$ C.) $\frac{1}{2} - \ln 2$ D.) $\ln 2$ E.) 1
F.) The integral diverges.

5. Evaluate the integral.

$$\int_0^{\pi/2} \sin^3(x)\,dx$$

A.) 0 B.) 1/3 C.) 1/2 D.) 2/3 E.) 1
F.) The integral diverges

6. Evaluate the integral

$$\int_0^1 \frac{1}{(x-2)(x-3)}\,dx$$

A.) $\ln \frac{1}{2}$ B.) $\ln \frac{2}{3}$ C.) $\ln \frac{3}{4}$ D.) $\ln \frac{4}{3}$ E.) $\ln \frac{3}{2}$ F.) $\ln 2$

7. Evaluate the integral

$$\int_{-3}^{3} \sqrt{9 - x^2}\, dx$$

A.) 0 B.) $\frac{99}{8}$ C.) $\frac{9\pi}{4}$ D.) $\frac{9\pi}{2}$ E.) 9π F.) 18π

8. Evaluate the integral

$$\int_1^e x^2 \ln x \, dx$$

A.) 0 B.) 1 C.) $\ln 2 - 1$ D.) $\frac{2}{9}e^2 + \frac{1}{9}$ E.) $\frac{2}{9}e^2 + \frac{1}{3}$ F.) $\frac{1}{3}e^2 - 1$

9. Evaluate the integral

$$\int_2^3 \frac{1}{x^2(x-1)}\,dx$$

A.) 1 B.) $\frac{9}{4}$ C.) $\ln 3$ D.) $4\ln 2 - 1$ E.) $4\ln 2 - 3\ln 3$ F.) $2\ln 2 - \ln 3 - \frac{1}{6}$

10. Evaluate the integral

$$\int_0^{\pi} e^x \sin(x)\, dx$$

A.) 1 B.) $\frac{1}{2}$ C.) $e^{\pi}+1$ D.) $\frac{\pi}{2}$ E.) $\frac{e^{\pi}+1}{2}$ F.) $\frac{e^{2\pi}+e^{\pi}+1}{2}$

11. Evaluate the integral

$$\int_0^4 \frac{1}{(9+x^2)^{3/2}}\,dx$$

A.) $\frac{14}{3}$ B.) $\frac{2}{3}\ln 3$ C.) $\frac{4}{3}\pi$ D.) 4.75 E.) 4/45 F.) 1

12. Evaluate the integral

$$\int_0^3 \frac{1}{(x-1)^3}\,dx$$

A.) $\frac{3}{8}$ B.) $\frac{1}{2}\ln 3$ C.) $\frac{9}{4}\pi$ D.) 0 E.) $\frac{8}{3}$ F.) The integral diverges.

Free Response 1.

Part (a):

Set up but do not evaluate an integral for the length of the curve $y = \ln(\cos(x))$ over the interval $0 \leq x \leq \pi/6$.

Part (b):

Set up but do not evaluate an integral for the surface area obtained by rotating the curve $y = \ln(\cos(x))$ over the interval $0 \leq x \leq \pi/6$ about the x-axis.

Part (c):

Set up but do not evaluate an integral for the surface area obtained by rotating the curve $y = \ln(\cos(x))$ over the interval $0 \leq x \leq \pi/6$ about the y-axis.

Free Response 2. Evaluate the integral

$$\int_0^{\pi} \sin^4(x)\, dx$$

Free Response 3. Suppose $p(x)$ is the **demand curve** for a product (i.e. $p(x)$ is price in dollars at which x units can be sold). Then the **consumer surplus** for a price P is

$$S = \int_0^A p(x) - P$$

(where $p(A) = P$).

Suppose Widget International just invented a new *Super Widget* such that the demand curve for super widgets is given by

$$p(x) = 64 \cdot (2^{-x}) \quad \text{where } p(x) \text{ is in \$}$$

What is the consumer surplus at a price of \$16?