

## (Math 170) Homework 4:

Due October 9, 2008

For a natural number  $n$  a multiplication table  $\pmod n$  consists is a grid such that

- The rows and columns are each labeled with numbers 0 to  $n - 1$
- The number in the  $a$ th column and the  $b$ th row is  $a \times b \pmod n$ .

For example, the multiplication tables  $\pmod{10}$  is the following:

<b>mod 10</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>	0	0	0	0	0	0	0	0	0	0
<b>1</b>	0	1	2	3	4	5	6	7	8	9
<b>2</b>	0	2	4	6	8	0	2	4	6	8
<b>3</b>	0	3	6	9	2	5	8	1	4	7
<b>4</b>	0	4	8	2	6	0	4	8	2	6
<b>5</b>	0	5	0	5	0	5	0	5	0	5
<b>6</b>	0	6	2	8	4	0	6	2	8	4
<b>7</b>	0	7	4	1	8	5	2	9	6	3
<b>8</b>	0	8	6	4	2	0	8	6	4	2
<b>9</b>	0	9	8	7	6	5	4	3	2	1

And the multiplication tables  $\pmod{9}$  is the following:

<b>mod 9</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
<b>0</b>	0	0	0	0	0	0	0	0	0
<b>1</b>	0	1	2	3	4	5	6	7	8
<b>2</b>	0	2	4	6	8	1	3	5	7
<b>3</b>	0	3	6	0	3	6	0	3	6
<b>4</b>	0	4	8	3	7	2	6	1	5
<b>5</b>	0	5	1	6	2	7	3	8	4
<b>6</b>	0	6	3	0	6	3	0	6	3
<b>7</b>	0	7	5	3	1	8	6	4	2
<b>8</b>	0	8	7	6	5	4	3	2	1

So given this table to find  $7 \times 4 \pmod{9}$  I look at the row labeled 7 and the column labeled 4 and find their intersection and I see the answer is 1.

Exercise 1: Write out a multiplication table mod 2, mod 3, mod 4, mod 5, mod 6, mod 7, mod 8.

Exercise 2: For each table in Exercise 1 and the two tables on this sheet find the pairs of numbers which are multiplicative inverses (i.e.  $a$  and  $b$  such that  $a \times b = 1 \pmod{n}$ ).

For which tables does every number have a multiplicative inverse? Do you think every number on the multiplicative table mod 11 will have an inverse? What about mod 12? Can you make any general predictions?

Exercise 3: Heart of Mathematics Chapter 2.4 Exercise 3

Exercise 4: Heart of Mathematics Chapter 2.4 Exercise 5

Exercise 5: Heart of Mathematics Chapter 2.4 Exercise 8

Exercise 6: Heart of Mathematics Chapter 2.4 Exercise 18

Exercise 7: Heart of Mathematics Chapter 2.4 Exercise 24

Exercise 8: Heart of Mathematics Chapter 2.4 Exercise 32

Exercise 9: Heart of Mathematics Chapter 2.4 Exercise 39