Math 104-006

Chapter 12.4: Comparison Tests

Outline For Today

- Comparison Tests
- Limits Comparison Tests
- Hierarchy of Functions

Tail End

- $\sum_{n=0}^{\infty} a_n$ converge if and only if
- $\sum_{n=m}^{\infty} a_n$ converges

Comparison Test

- Suppose $\sum_{n=0}^{\infty} a_n, \sum_{n=0}^{\infty} b_n$ are series with

$0 \leq a_n \leq b_n$ for all n.

- (i) If $\sum_{n=0}^{\infty} b_n$ is convergent then so is $\sum_{n=0}^{\infty} a_n$
- (ii) If $\sum_{n=0}^{\infty} a_n$ is divergent then so is $\sum_{n=0}^{\infty} b_n$

Example

- Does $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge?

Example

- Does $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge? YES.
- If n > 1 then $\frac{1}{n^n} < \frac{1}{n^2}$ and

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (because it is a p-series with p > 1)

So $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converges!!

Try An Example

Does $$\sum_{n=1}^{\infty} \frac{2n+1}{2n^3+n}$$ converge?

A) YES

B) NO

C) None of the above

Try An Example

Does $$\sum_{n=1}^{\infty} \frac{2n+1}{2n^3+n}$$ converge?

A) YES

B) NO

C) None of the above

Limit Comparison Test

- Suppose $\sum_{n=0}^{\infty} a_n, \sum_{n=0}^{\infty} b_n$ are series with positive terms
- If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c > 0 is a finite number
- (i) If $\sum_{n=0}^{\infty} a_n$ converges then so does $\sum_{n=0}^{\infty} b_n$
- (ii) If $\sum_{n=0}^{\infty} a_n$ is diverges then so does $\sum_{n=0}^{\infty} b_n$

Example

Does $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converge?

$$\lim_{n\to\infty} \frac{\frac{1}{2^n}}{\frac{1}{2^n - 1}} = \lim_{n\to\infty} \frac{2^n - 1}{2^n} = \lim_{n\to\infty} 1 - \frac{1}{2^n} = 1$$

So by the limit comparison test $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

converges if and only if $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges

Example Continued

But we know that $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges and so

$\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ converges by the limit comparison test

You try one

Does $\sum_{n=2}^{\infty} \frac{n^2 - n - 1}{100n^3 + n}$ converge?

A) YES

B) NO

C) None of the above

You try one

Does $\sum_{n=2}^{\infty} \frac{n^2 - n - 1}{100n^3 + n}$ converge?

A) YES

B) NO

C) None of the above

Rate of Growth

There is a list of types of functions in terms of “rate” of growth. This is:

constant	i.e.	1, 7 or -2
logarithms	i.e.	$Log_2(x)$ or $ln(2x+1)$
polynomials	i.e.	x^2+2 or $x^{100}-x^{10}+1$
exponentials	i.e.	2^x or $e^{\sqrt{x}}$
factorials	i.e.	$n!$ or $(2n+1)!$

Rate of Growth Continued

When considering whether or not a series converges it suffices to consider only the fastest growing functions.

You try one

Does $\sum_{n=100}^{\infty} \frac{3^n - n^{100} - \log(n)}{n! - 2^n + n}$ converge?

A) YES

B) NO

C) None of the above

You try one

Does $\sum_{n=100}^{\infty} \frac{3^n - n^{100} - \log(n)}{n! - 2^n + n}$ converge?

A) YES

B) NO

C) None of the above