Math 104-006

Chapter 8.5: Strategies for Integration

Outline For Today

- General Integration Strategy

Step 1

Simplify the Integrand if possible.

Sometimes the use of algebraic manipulation or trigonometric identities will simplify the integrand and make the method of integration obvious

Example

$$\int \frac{\tan(x)}{\sec^2(x)}dx = \int \frac{\sin(x)}{\cos(x)}\cos^2(x)dx$$

$$= \int \sin(x)\cos(x)dx$$

$$= \frac{1}{2}\int \sin(2x)dx$$

$$= -\frac{1}{4}\cos(2x) + C$$

Step 2

Look for an Obvious Substitution.

Try to find some function $u = g(x)$ in the integrand whose differential $du=g'(x)dx$ also occurs, apart from a constant factor

Step 3

Classify the Integral According to Its Form.

(a) ***Trigonometric Functions:*** If f(x) is a product of powers of sin(x) and cos(x) of tan(x) and sec(x) or of cot(x) and csc(x) then we use the substitutions of 8.2

(b) ***Rational Functions:*** If f is a rational function we use the procedure of Section 8.4 involving partial fractions

Step 3 Continued

(c) ***Integration by Parts:*** If f(x) is a product of a power of x (or a polynomial) and a transcendental function (such as a trigonometric, exponential, or logarithmic function) then we try integration by parts, choosing u and dv according to advice given in Section 8.1

Step 3 Continued

(d) Radicals: Particular kinds of substitutions are recommended when certain radicals appear

(i) If $\sqrt{\pm x^2 \pm a^2}$ occurs we use a trigonometric substitution according to the table in Section 8.3

(ii) If $\sqrt[n]{ax+b}$ occurs we use the rationalizing substitution $u = \sqrt[n]{ax+b}$

More generally, this sometimes works for $u = \sqrt[n]{g(x)}$

Step 4

Try Again.

If the first three steps have not produced the answer, remember that there are basically only two methods of integration: Substitution and By Parts

(a) Try Substitution: Even if no substitution is obvious some inspiration or ingenuity (or even desperation) may suggest an appropriate substitution.

Step 4 Continued

(b)Try Parts: Although integration by parts is used most of the time on products of the form described in Step 3(c) it is sometimes effective on single functions. Looking at Section 8.1, we see that it works on $\tan^{-1}(x)$, $\sin^{-1}(x)$ and $\ln(x)$ and these are all inverse functions.

Step 4 Continued

(c) Manipulate the Integrand: Algebraic Manipulations (perhaps rationalizing the denominator or using trigonometric identities) may be useful in transforming the integral into an easier form. These may be more substantial than in Step 1 and may involve some ingenuity.

Example

$$\int \frac{dx}{1-\cos(x)} = \int \frac{1}{1-\cos(x)} \frac{1+\cos(x)}{1+\cos(x)} dx$$

$$= \int \frac{1+\cos(x)}{1-\cos^2(x)} dx$$

$$= \int \frac{1+\cos(x)}{\sin^2(x)} dx$$

$$= \int \frac{1}{\sin^2(x)} dx + \int \frac{\cos(x)}{\sin^2(x)} dx$$

$$= \int \csc^2(x) dx + \int \frac{\cos(x)}{\sin^2(x)} dx$$

Step 4 Continued

(d) Relate the Problem to Previous Problems: When you have built up some experience in integration, you may be able to ues a method on a given integral that is similar to a method you have already used on a previous integral. Or you may even be able to express the given integral in terms of a previous one.

Example

For example $\int \tan^2(x)\sec(x)dx$ is a hard integral.
But if we use the identity $\tan^2(x) = \sec^2(x) - 1$

then

$$\int \tan^2(x)\sec(x)dx \quad = \quad \int \sec^3(x) - \sec(x)dx$$

and we know what $\int \sec(x)dx$ and $\int \sec^3(x)dx$ are.

Step 4 Continued

(e) Use Several Methods: Sometimes two or three methods are required to evaluate an integral. The evaluation could involve several successive substitutions of different types, or it might combine integration by parts with one or more substitutions.