Math 104-006

Chapter 8.1: Integration by Parts

Outline For Today

- Integration By Parts

Product Rule of Derivatives

$d/dx\ [f(x)g(x) = f(x)g'(x) + g(x)f'(x)$

So

$d/dx\ [f(x)g(x) - g(x)f'(x) = f(x)g'(x)$

Integration By Parts

Taking the integral of both sides we get

$$\int [f'(x)g(x) + g'(x)f(x)]dx = f(x)g(x)$$

or equivalently

$$\int f'(x)g(x)dx + \int g'(x)f(x)dx = f(x)g(x)$$

Integration By Parts

Rearranging the equation we get

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

and if we let u = f(x) and v = g(x) then we have

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

because dv = g’(x)dx and du = f’(x)dx

Example

Lets find $\int x\cos(x)dx$ using integration by parts.

If we let:

u = x dv = cos(x) dx

du = dx v = sin(x)

Then $\int u \cdot dv = u \cdot v - \int v \cdot du$ or equivalently...

Example Continued

$$\int x \cdot \cos(x)dx = x \cdot \sin(x) - \int \sin(x)dx$$

$$= x \cdot \sin(x) - (-\cos(x)) + C$$

$$= x \cdot \sin(x) + \cos(x) + C$$

Now You Try One

What is $\int x^2 \cdot e^{-x} dx$?

A) $(x^2 + 2x + 2)e^{-x} + C$

B) $-(x^2 + 2)e^{-x} + C$

C) $(x^2 + 2x)e^{-x} + C$

D) $-(x^2 + 2x + 1)e^{-x} + C$

E) $(x^2 + 3x + 2)e^{-x} + C$

F) $-(x^2 + 2x + 2)e^{-x} + C$

Answer

What is $\int x^2 \cdot e^{-x} dx$?

A) $(x^2+2x+2)e^{-x}+C$

B) $-(x^2+2)e^{-x}+C$

C) $(x^2+2x)e^{-x}+C$

D) $-(x^2+2x+1)e^{-x}+C$

E) $(x^2+3x+2)e^{-x}+C$

F) $-(x^2+2x+2)e^{-x}+C$

Try A Harder One

What is $\int \ln(x)dx$?

A) $x\ln(x)+x+C$

B) $x\ln(x)-x+C$

C) $(\ln(x))^2/2+C$

D) $-(\ln(x))^2+x+C$

E) $2\ln(x)+x+C$

F) $\ln(x)-2x+C$

Try A Harder One

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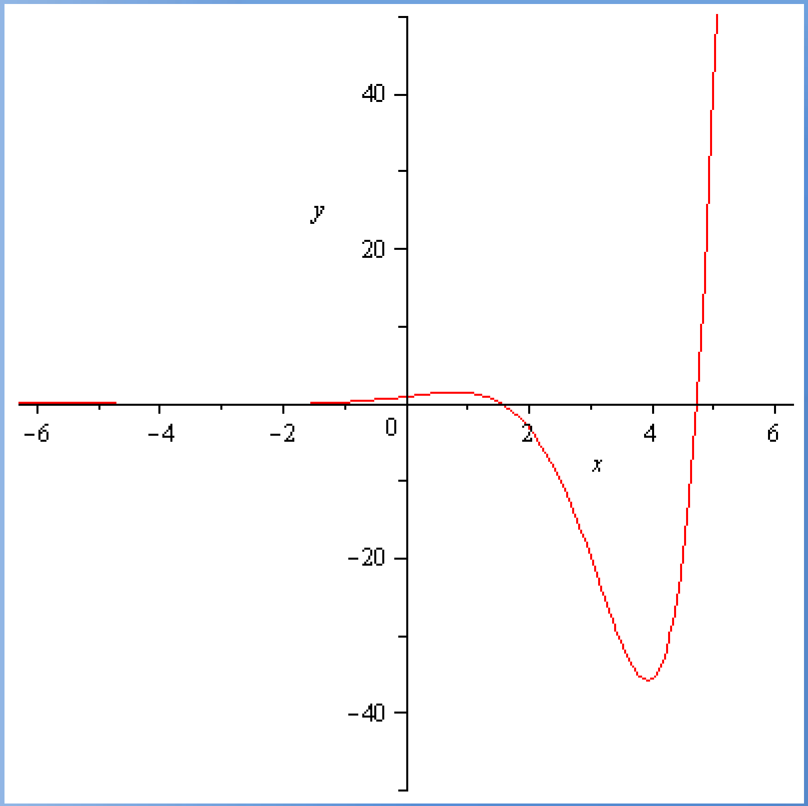
F) $\ln(x)-2x+C$

Another Example

What is $\int e^x \cos(x)dx$?



Answer

We can let

$$u = e^x \qquad dv = \cos(x)dx$$

$$du = e^x dx \qquad v = \sin(x)$$

Then we have

$$\int e^x \cos(x)dx = e^x \sin(x) - \int e^x \sin(x)dx$$

Answer Cont.

Now we can let

$$u = e^x \qquad dv = \sin(x)dx$$

$$du = e^x dx \qquad v = -\cos(x)$$

Then we have

$$\int e^x \sin(x)dx = -e^x \cos(x) + \int e^x \cos(x)dx$$

Answer Cont.

So we have

$$\begin{aligned}\int e^x\cos(x)dx &= e^x\sin(x) - (-e^x\cos(x) + \int e^x\cos(x)dx) \\ &= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)dx\end{aligned}$$

and hence

$$2\int e^x\cos(x)dx = e^x\sin(x) - e^x\cos(x)$$

Or equivalently

$$\int e^x\cos(x)dx = \frac{1}{2}(e^x\sin(x) + e^x\cos(x))$$

Now You Try One

What is $\int_0^1 \sin^{-1}(x)dx$?

A) -1- $\pi/2$

B) 1- $\pi/2$

C) -2+ π

D) -1+ $\pi/2$

E) -1+ π

F) 2- $\pi/2$

Answer

What is $\int_0^1 \sin^{-1}(x)dx$?

A) -1- $\pi/2$

B) 1- $\pi/2$

C) -2+ π

D) -1+ $\pi/2$

E) -1+ π

F) 2- $\pi/2$