Math 104-006

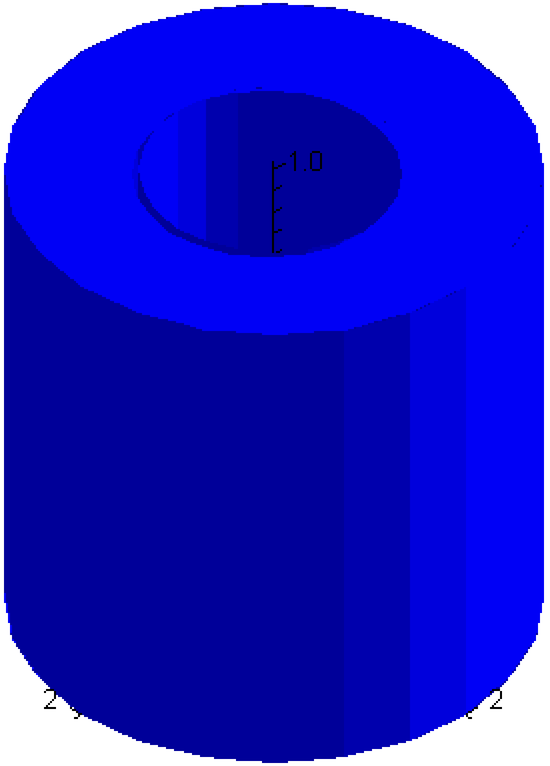
Chapter 6.3: Volumes by Cylindrical Shells

Outline For Today

- Approximations to Volumes Using Shells
- Method of Shells for Calculating Volume of Curves Rotated Around an Axis

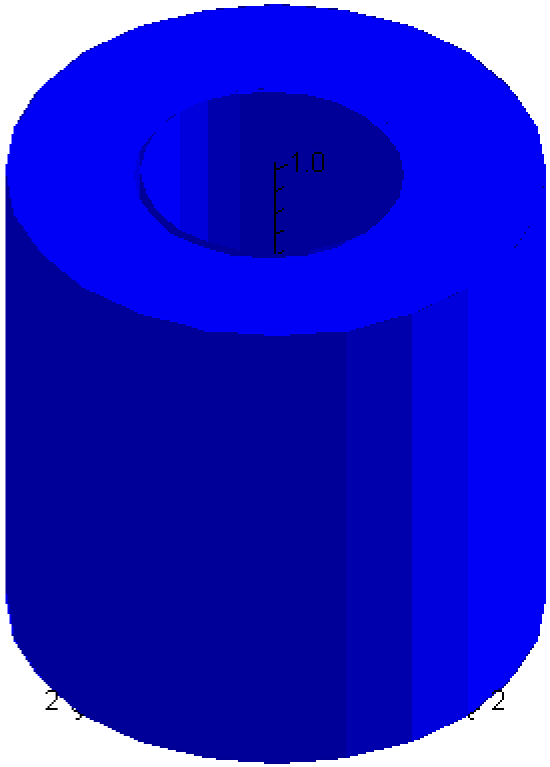
Volume of a Shell

- We want to find the volume of a single box rotated around the y-axis.



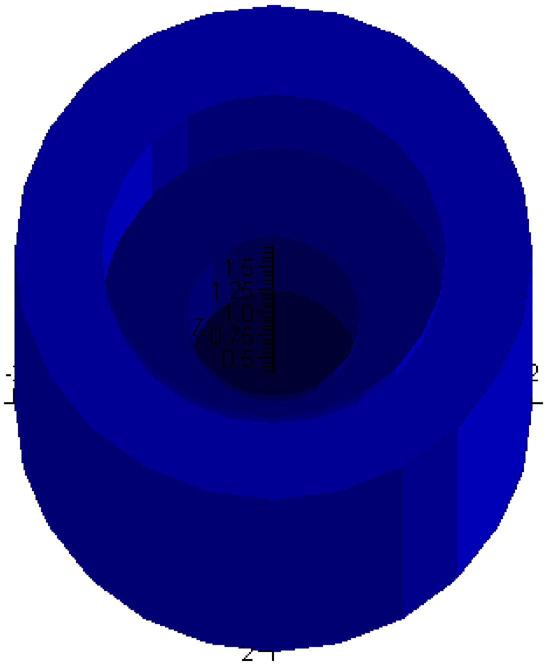
Volume of Box Rotated Around y-axis

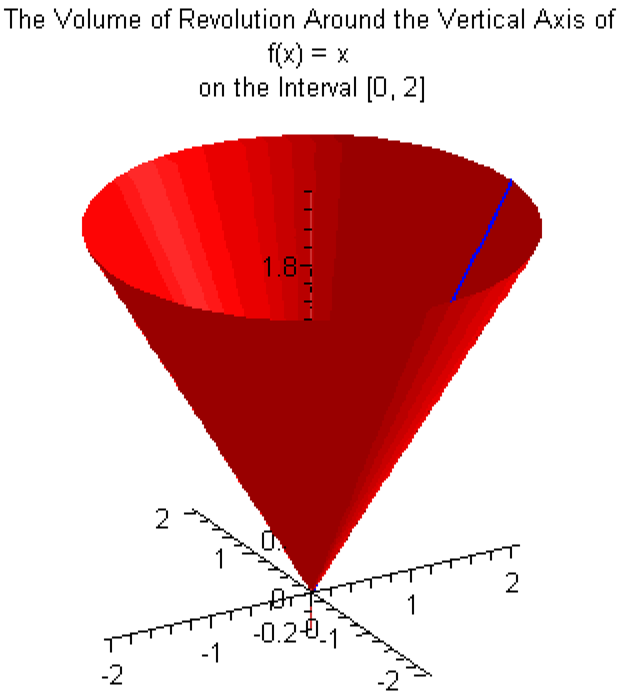
Volume Of Shell = $2\pi x \cdot f(x) \cdot \Delta x$



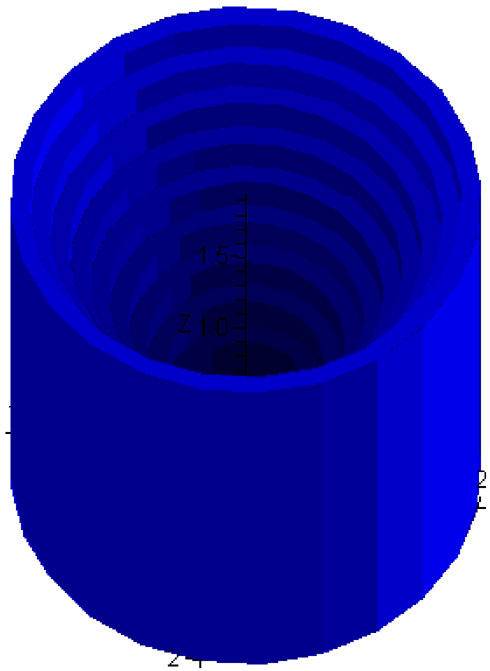
Approximate Area of y=f(x) Rotated About y-axis

$$\text{Approximate Volume} = \sum_{i=0}^{n} 2\pi x \cdot f(x) \cdot \Delta x$$

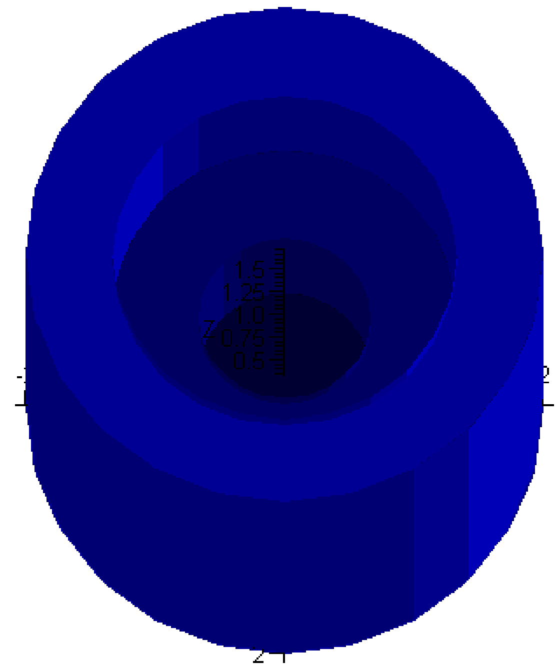




The More Shells We Use The Better The Approximation



12 cylinders



3 cylinders

Actual Volume

- As the number of shells goes to infinity the error in the approximation goes to zero.

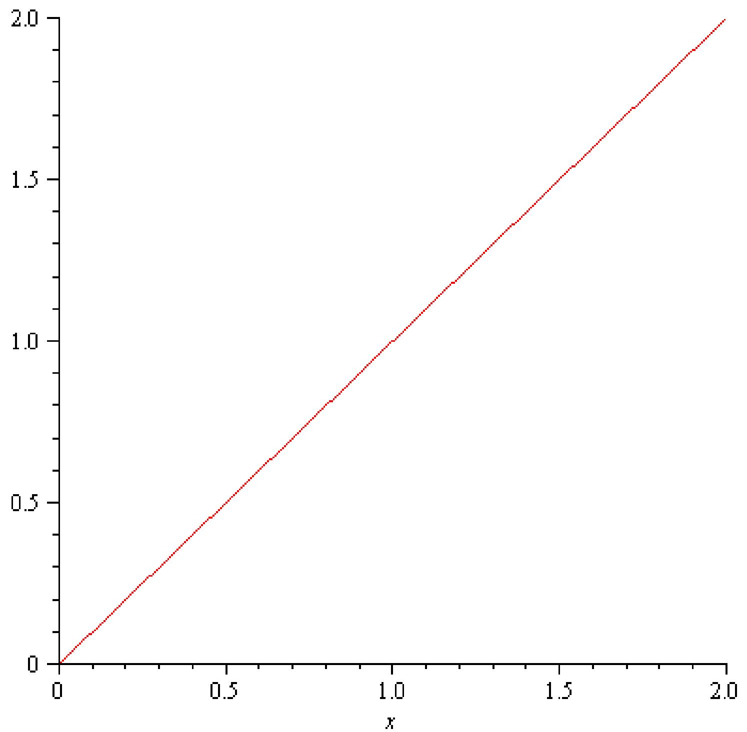
Volume = $\lim_{n\to\infty}\sum_{i=0}^{n} 2\pi x \cdot f(x) \cdot \Delta x$

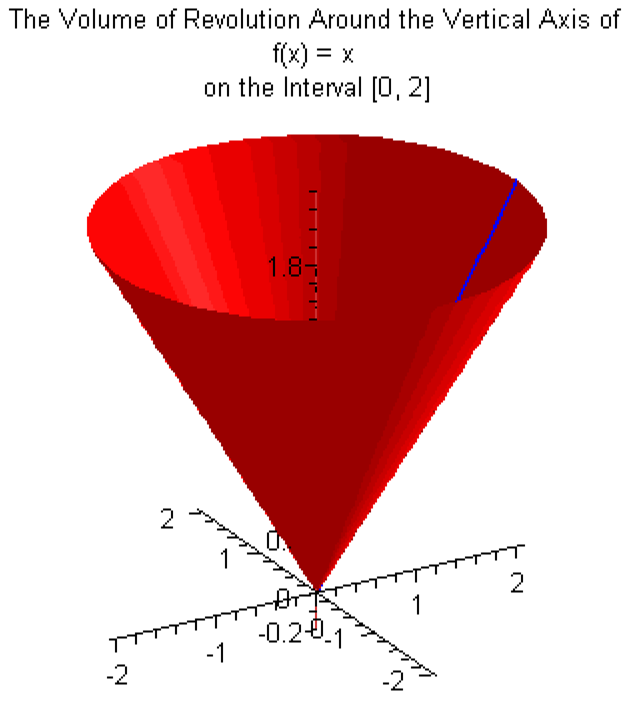
= $\int_a^b 2\pi x \cdot f(x) \cdot dx$

Volume of Example f(x) = x Rotated Around y-axis (from x = 0 to 2)

Volume

$$V(x) = \int_0^2 2\pi x \cdot x\,dx = \pi \cdot x^2 \Big|_0^2 = 4\pi$$





Your Turn

What is the volume of the shape obtained by rotating the area between the x and y axis and the curve $f(x) = 4-x^2$ about the y-axis?

A) $15\pi/2$

B) 8π

C) $16/3$

D) $16\pi/3$

E) $8\pi/3$

F) 12

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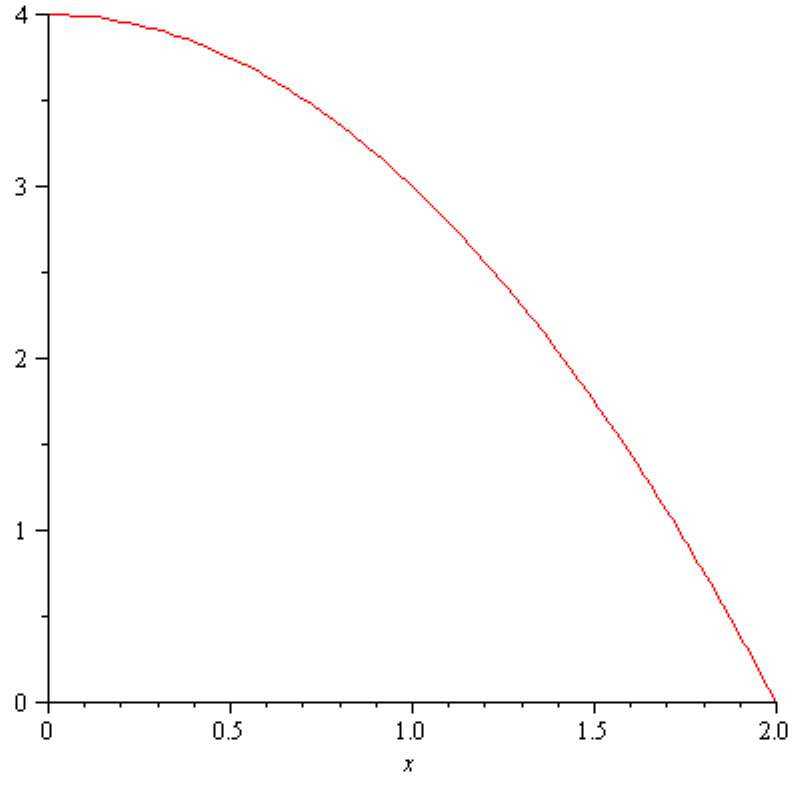
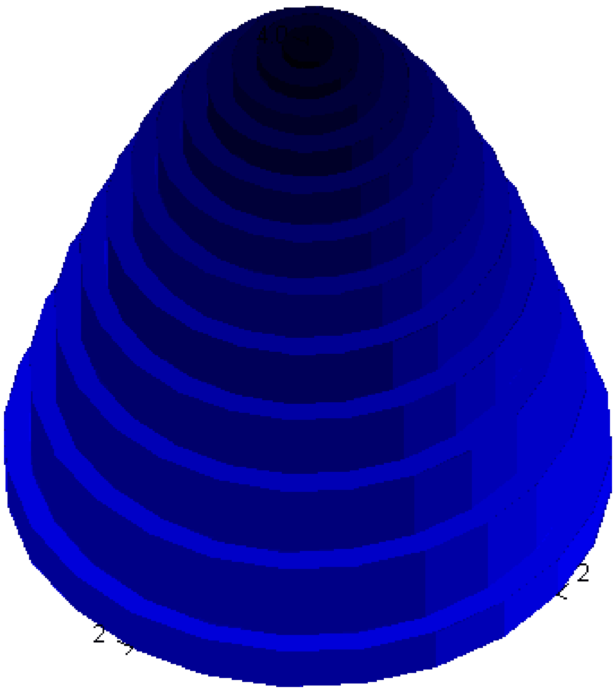
F) 12

Proof

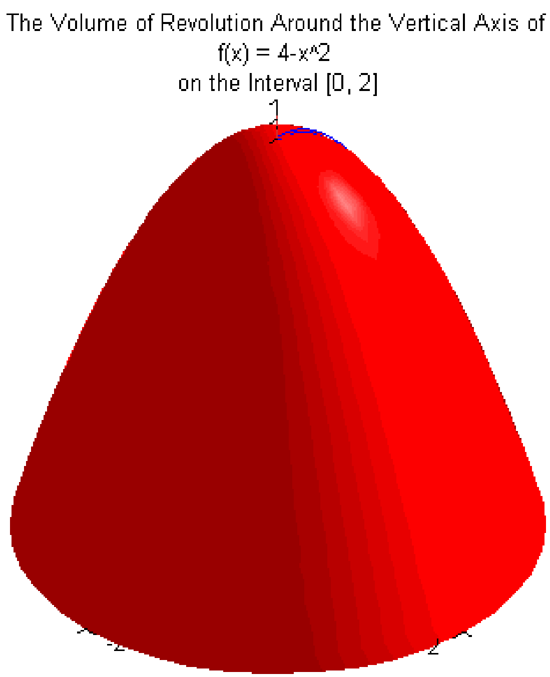
The Left Bound is x=0 and the right bound is where f(x) = 0 or x = 2

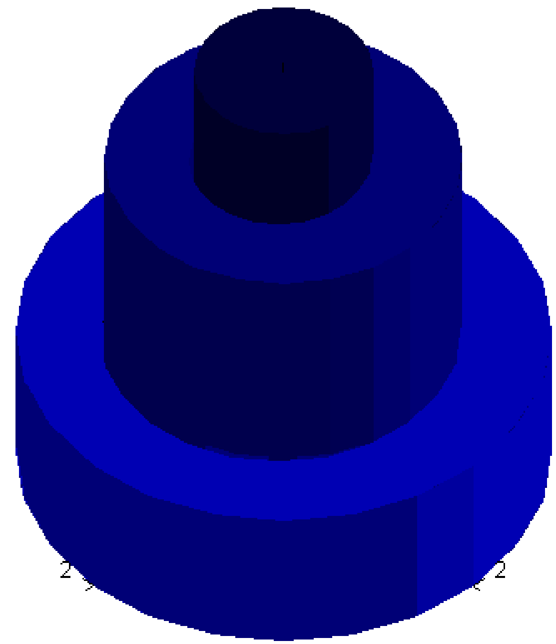


$$\text{Volume} = \int_0^2 2\pi x \cdot x\,dx = 2\pi \cdot x^3/3\Big|_0^2 = 16\pi/3$$



12 Washers





3 Washers

Lets Try Another One

Suppose we rotate the region enclosed by

$$f(x) = x^2, g(x) = x$$

about the line x = -1. What is the volume?

A) 2π

B) π

C) 2

D) $\pi/4$

E) 4π

F) $\pi/2$

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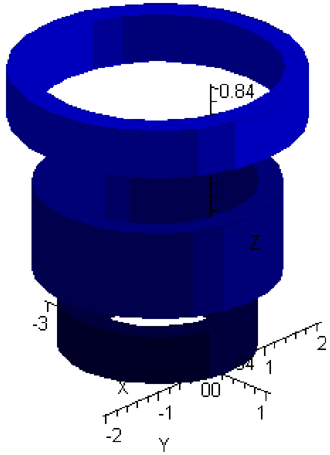
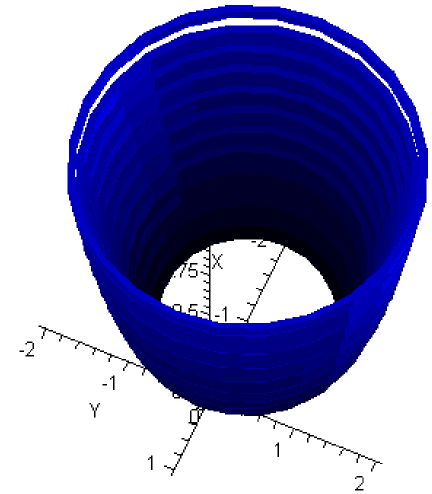
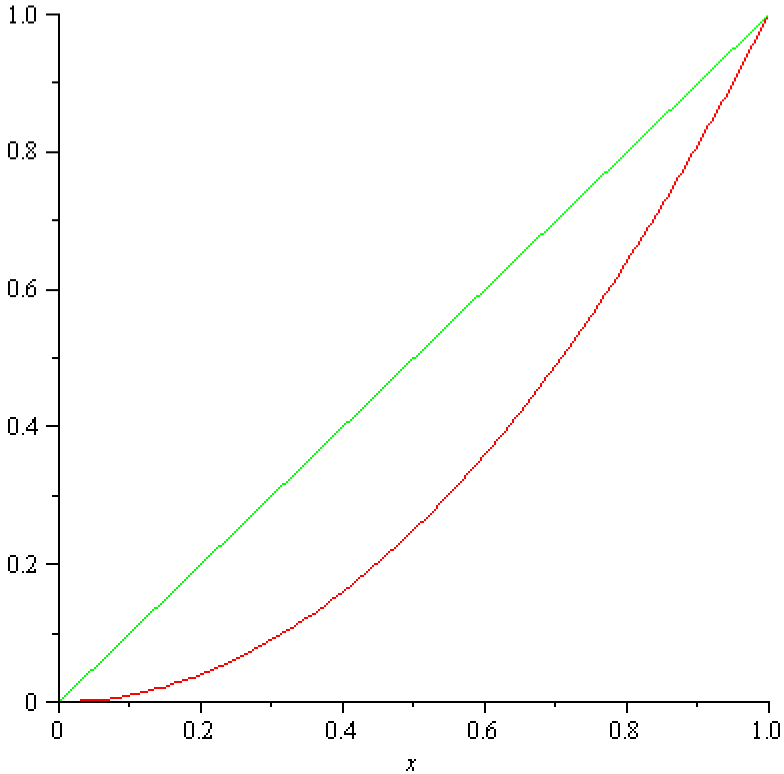
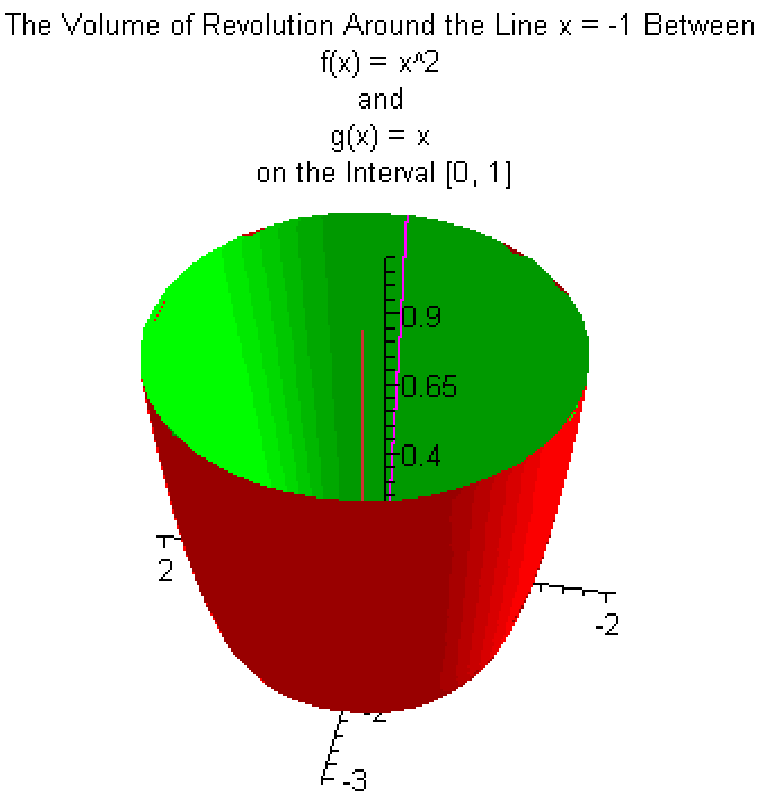
Picture

The shape looks like









Volume

The Volume is:

Volume(x) = $$\int_0^1 2\pi \cdot (x+1)[x - x^2]dx$$

$$= 2\pi \cdot [x^2/2 - x^4/4]\Big|_0^1$$

$$= 2\pi \cdot [1/2 - 1/4] = \pi/2$$

Method of Shells

If f(x) is non-negate on (a, b) then the volume obtained by rotating f(x) about the x = c axis is:

$$\text{Volume(x)} = \int_a^b 2\pi \cdot (x-c) \cdot f(x)dx$$

This is called the “Method of Shells” because the approximation to the volume is obtained by adding the volume of shells.