Math 104-006

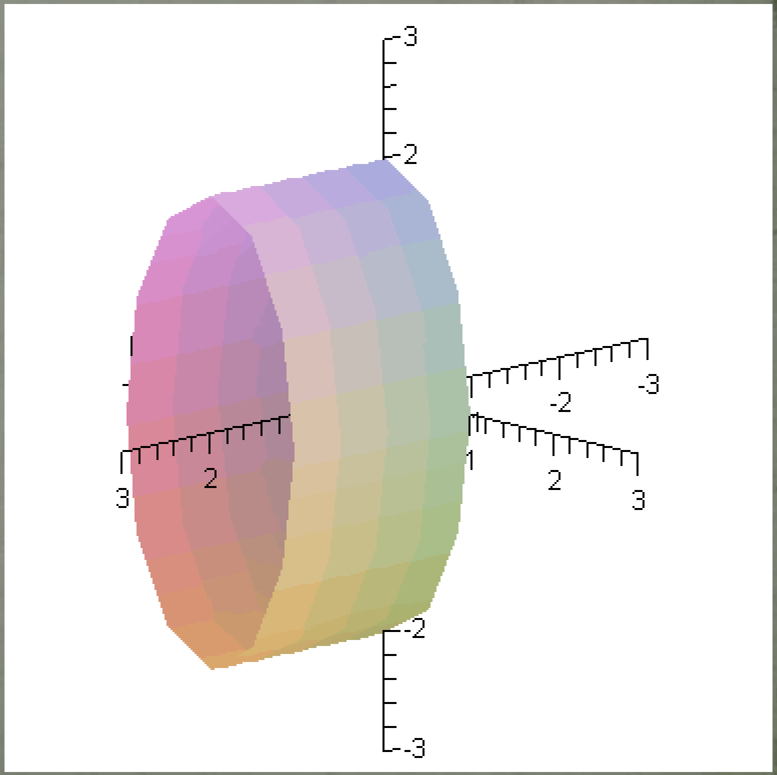
Chapter 6.2: Volume

Outline For Today

- Approximations to Volumes Using Cylinders
- Method of Disks for Calculating Volume of Curves Rotated Around an Axis

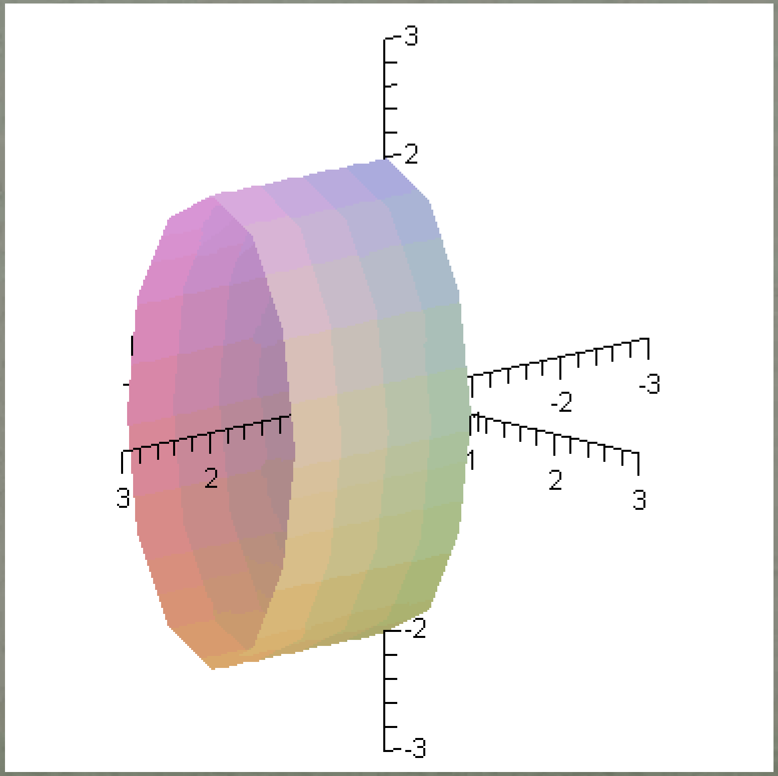
Volume of a Cylinder

- We want to find the volume of a single cylinder with irregular base.



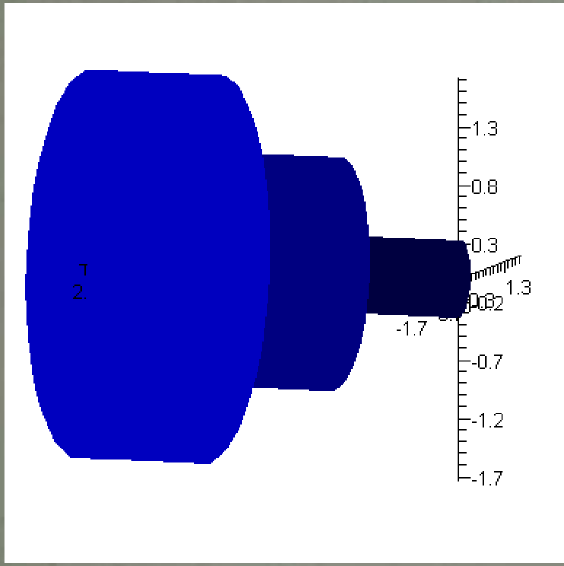
Volume of Cylinder

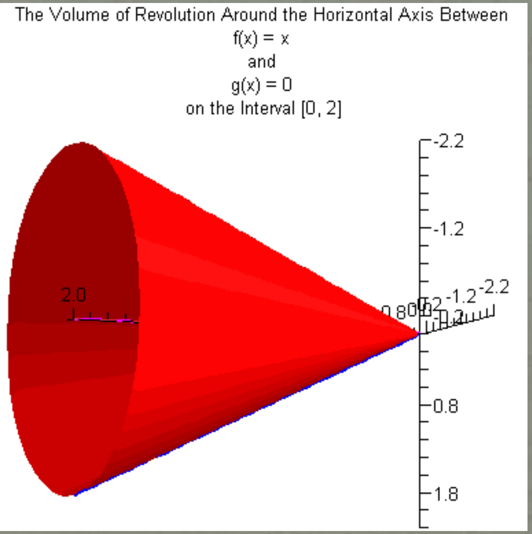
Volume = Base Area * Height

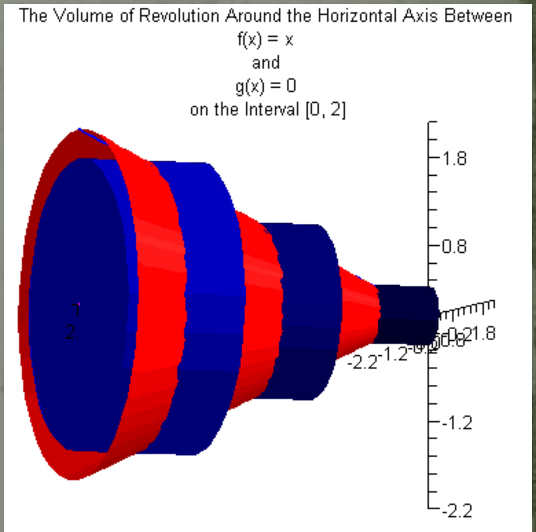


Approximate Volume

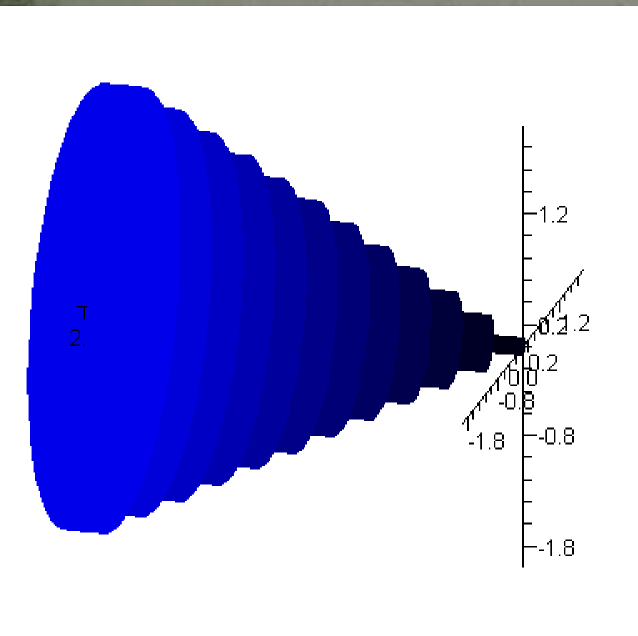
- We can approximate the volume of a shape by using several cylinders



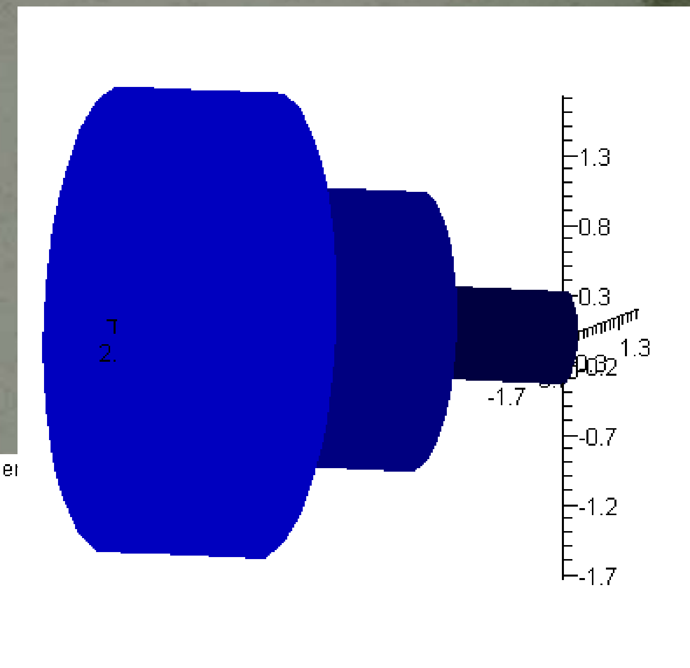




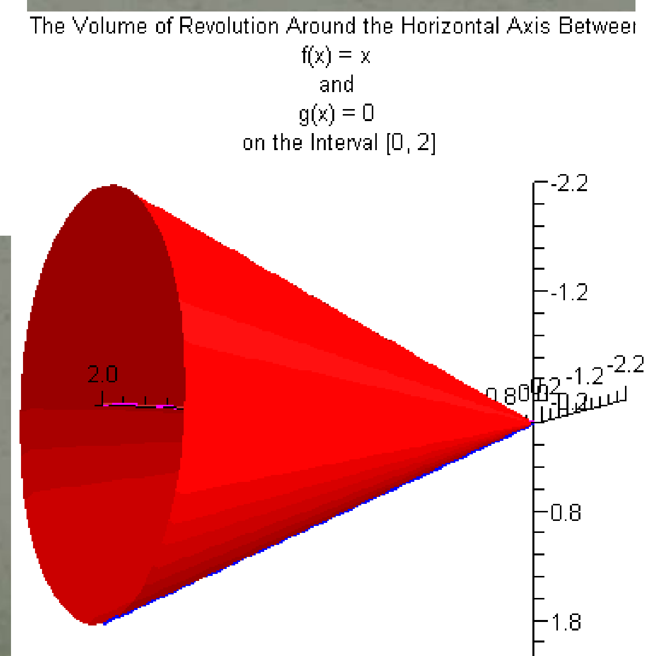
The More Cylinders We Use The Better The Approximation



12 cylinders



3 cylinders



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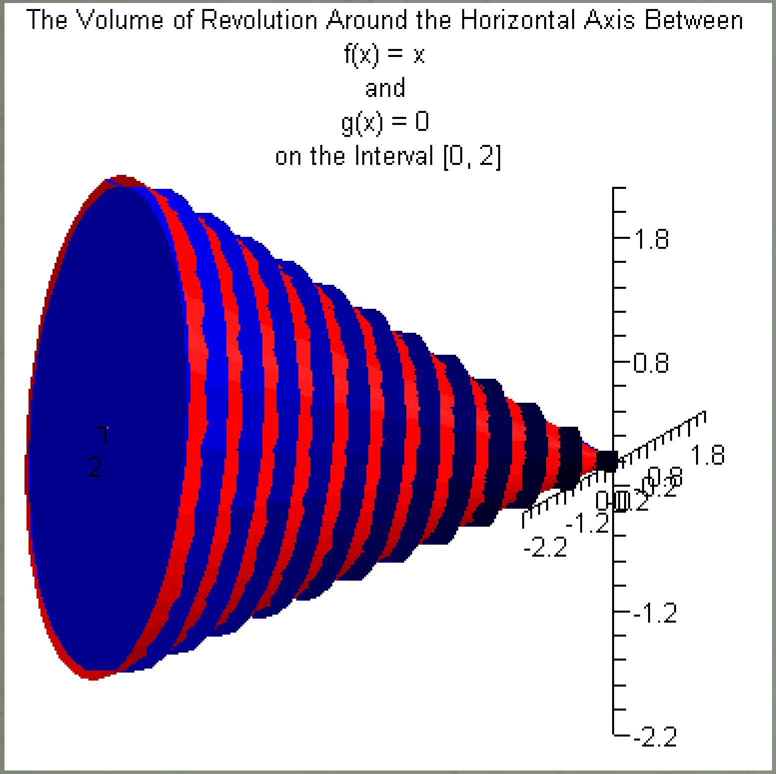
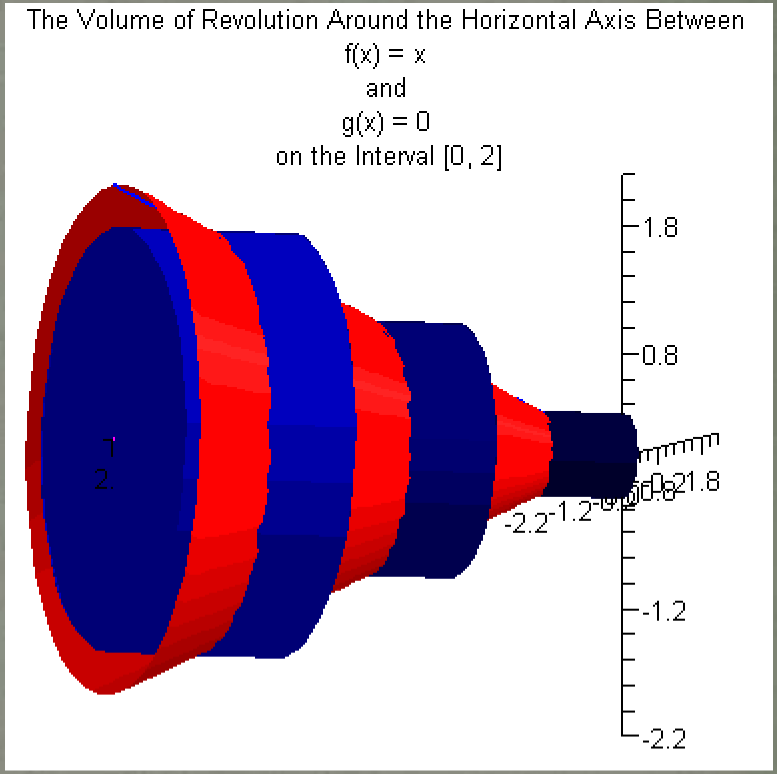


12 cylinders



3 cylinders

Volume of Approximation

If A(x) is the cross section area at x then an approximation to the volume is the sum of all the cylinders.

$$\text{Approximate Volume} = \sum_{i=0}^{n} A(x_i^*)\Delta x$$

Actual Volume

- As the number of cylinders goes to infinity the error in the approximation goes to zero.

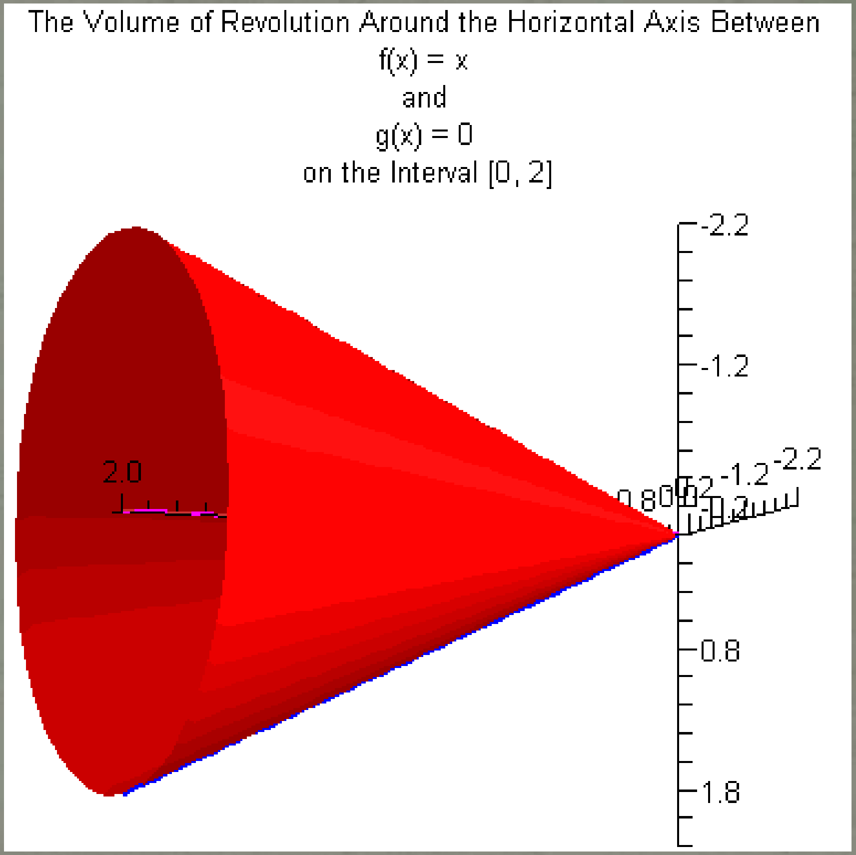
$$\text{Area} = \lim_{n\to\infty} \sum_{i=0}^{n} A(x_i^*)\Delta x$$

$$\text{Area} = \int_a^b A(x)\,dx$$

Volume of Example

Cross Section Area

$$A(x) = \pi \cdot x^2$$



Volume

$$V(x) = \int_0^2 \pi \cdot x^2 dx = \pi \cdot x^3 / 3 \Big|_0^2 = \pi \cdot 8/3$$

Your Turn

Suppose there is a prism 2 meters high whose cross sectional x meters from the ground is a square with side x. What is its volume?

A) $\pi/2\ m^3$

B) $8/3\ \pi\ m^3$

C) $8/3\ m^3$

D) $8\ m^3$

E) $8/3\ m^2$

F) $4/3\ m^3$

Answer

Suppose there is a prism 2 meters high whose cross sectional x meters from the ground is a square with side x. What is its volume?

A) $\pi/2\ m^3$

B) $8/3\ \pi\ m^3$

C) $8/3\ m^3$

D) $8\ m^3$

E) $8/3\ m^2$

F) $4/3\ m^3$

Lets Try Another One

Suppose we rotate the curve $y = x^2$ from x=0 to x=2 about the x-axis. What is the volume of the resulting shape?

A) 6π

B) $8/3\ \pi$

C) 32

D) $16/5\ \pi$

E) $32/5\ \pi$

F) 16/5

Answer

Suppose we rotate the curve $y = x^2$ from x=0 to x=2 about the x-axis. What is the volume of the resulting shape?

A) 6π

B) 8/3 π

C) 32

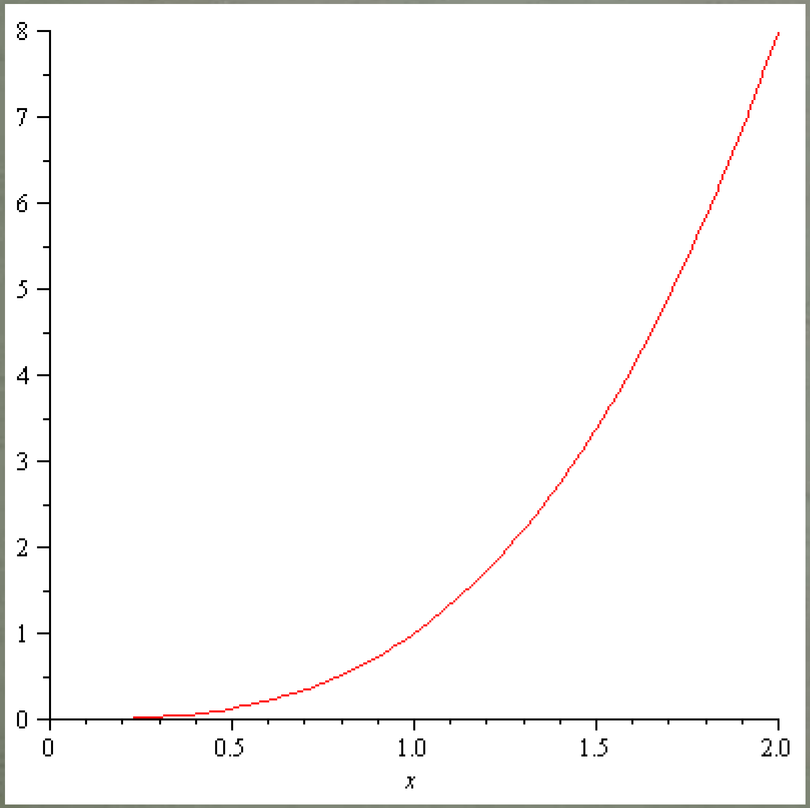
D) 16/5 π

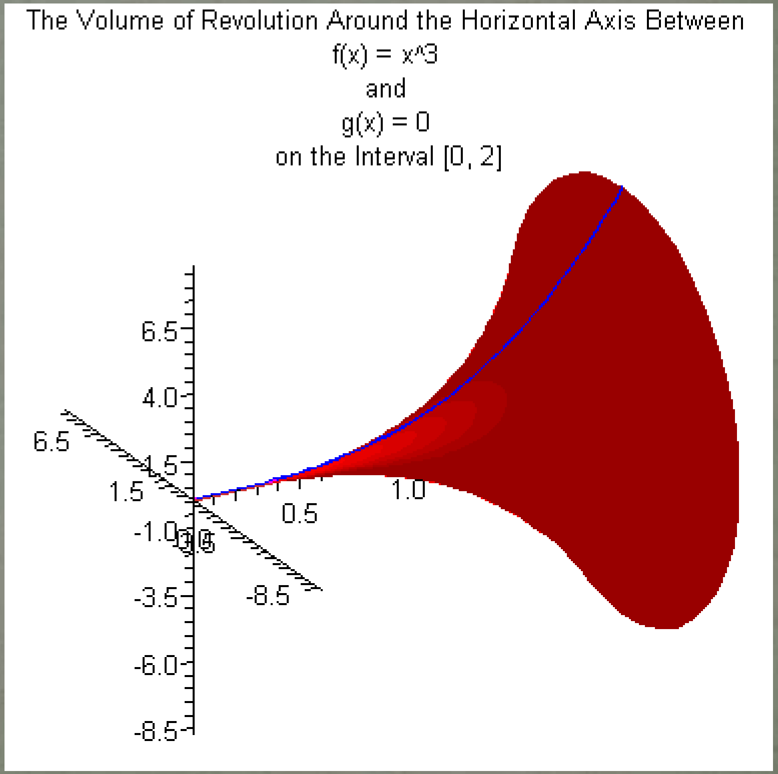
E) 32/5 π

F) 16/5

Picture

The shape looks like





Method of Disks

If f(x) is non-negate on (a, b) then the volume obtained by rotating f(x) about the x axis is:

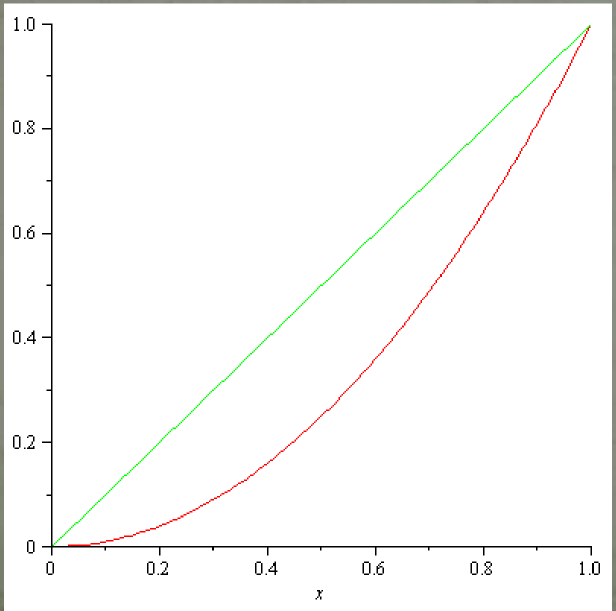
$$\text{Volume}(x) = \int_a^b \pi \cdot f(x)^2 \, dx$$

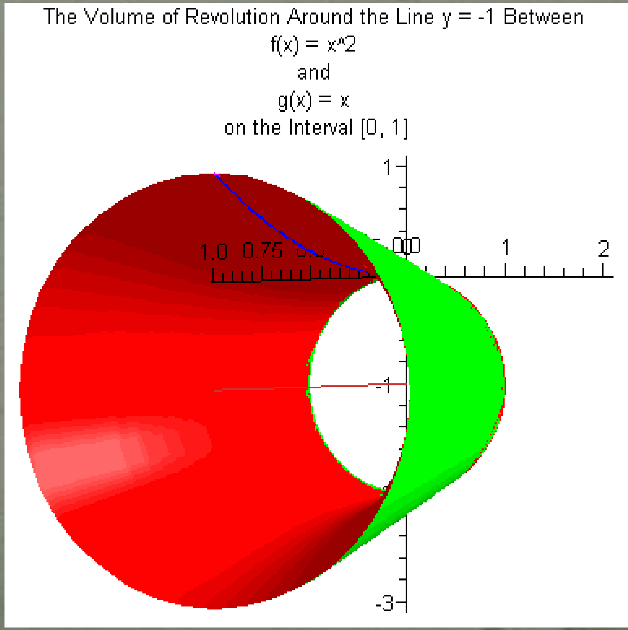
This is called the “Method of Disks” because the approximation to the volume is obtained by adding the volume of disks.

What is the volume obtained by rotating the region enclosed by the curves

$$f(x) = x^2, g(x) = x$$

about the line y = -1?





Cross Section Area

The area of a cross section at a x-coordinate x is:

$$\text{Area(x)} = \pi \cdot (f(x)+1)^2 - \pi \cdot (g(x)+1)^2$$

$$= \pi \cdot [(x+1)^2 - (x^2+1)^2]$$

$$= \pi \cdot [x^2 + 2x + 1 - (x^4 + 2x^2 + 1)]$$

$$= \pi \cdot [-x^4 - x^2 + 2x]$$

Volume

The Volume is:

Volume(x) = $\int_0^1 \pi \cdot [-x^4 - x^2 + 2x]dx$

$$= \pi \cdot [-x^5/5 - x^3/3 + x^2]\Big|_0^1$$

$$= \pi \cdot [-1/5 - 1/3 + 1] = \pi \cdot 7/15$$

General Method of Disks

- Find the outer radius (i.e. the distance of to the outer curve from the axis of rotation)
- Find the inner radius (i.e. the distance of to the inner curve from the axis of rotation)
- Volume is

$$\int_a^b \pi \cdot [r_{outer}{}^2 - r_{inner}{}^2]dx$$