Math 104-006

Chapter 10.2: Direction Fields and Euler’s Method

Outline For Today

- Direction Fields
- Euler’s Method

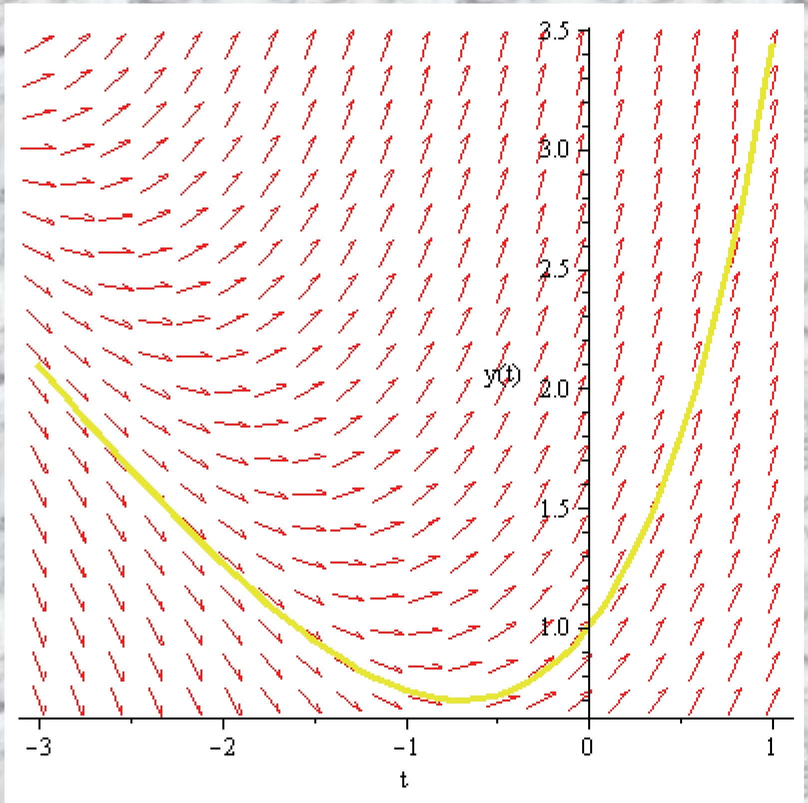
$y' = x + y \quad y(0) = 1$

- Given a first order differential equation as above we know what the slope of the tangent line is at any given point on the plane.

- If we draw small line segments at various points of the appropriate slope this will let us get an idea of what the curve looks like.

y'=x+y   y(0)=1 continued

- We then get a graph which looks like



- The gold curve is the curve we get by setting y(0) = 1

General Direction Fields

- In general suppose we have y’=F(x,y) where F(x,y) is some expansion of x and y.

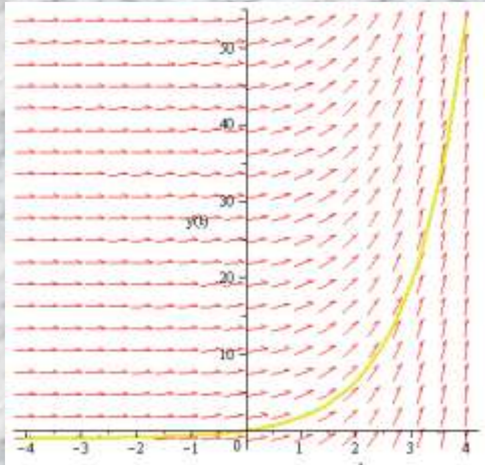
- If we draw short line segments at various points (x,y) with slopes F(x,y) the result is called a **direction field** (or **slope field**)

Try An Example

- What is the direction field of the equation

$$\frac{dy}{dx} = \sin(x)$$ with initial value y(0) = 0?

A)



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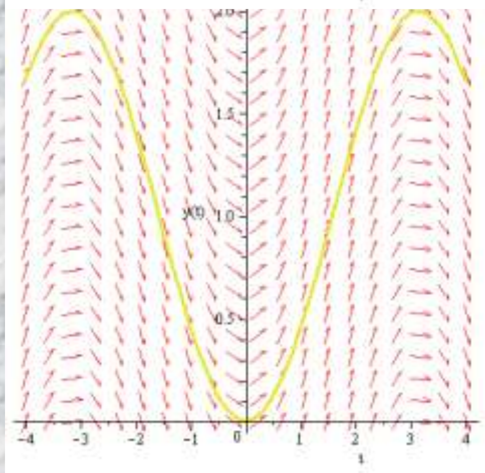

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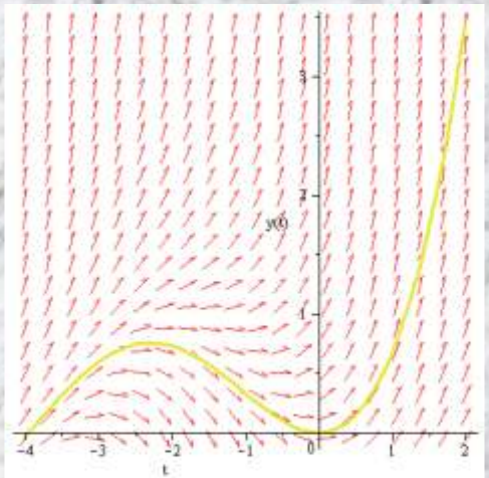

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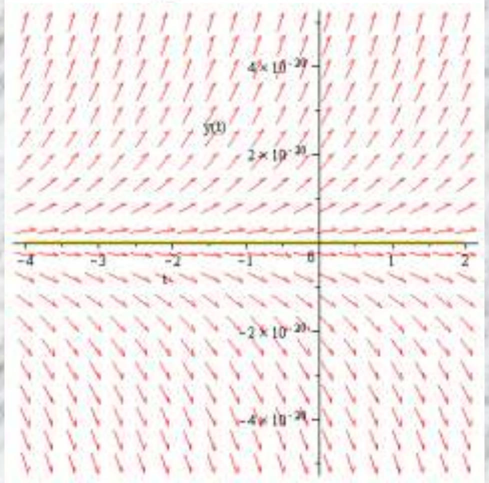

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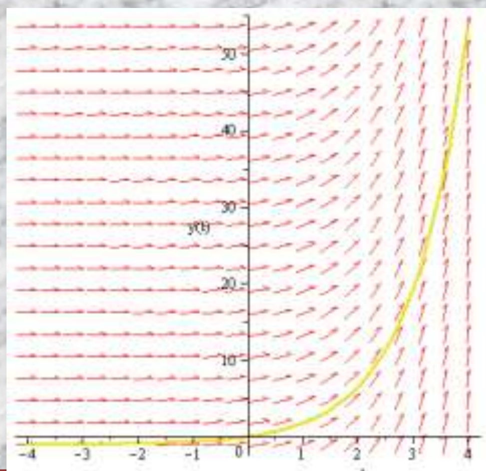

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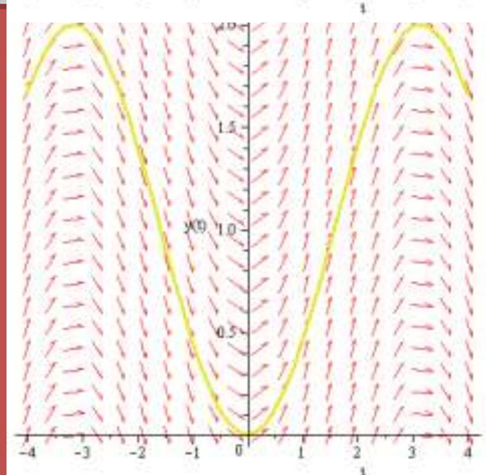

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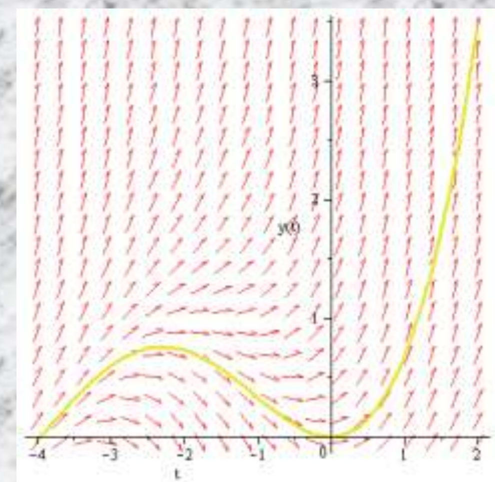

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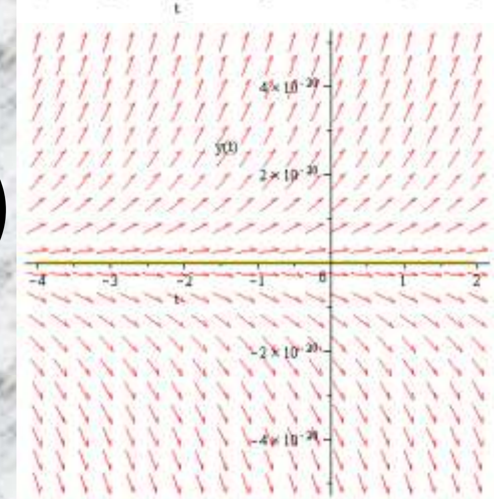

Try Another Example

- What is the direction field of the equation $\frac{dy}{dx} = y^2$ with initial value y(0) = 1?

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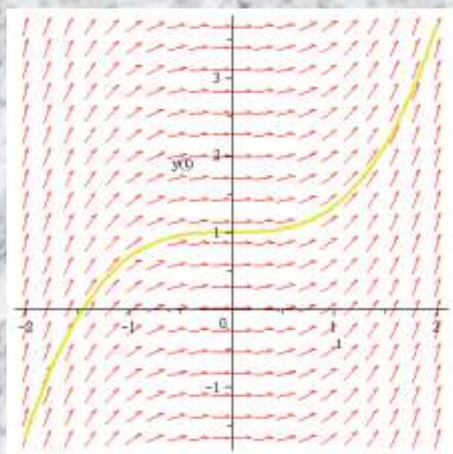

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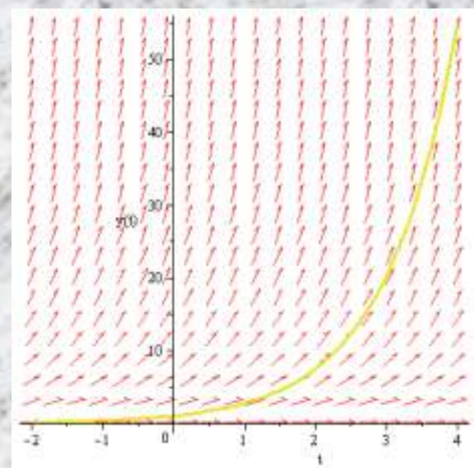

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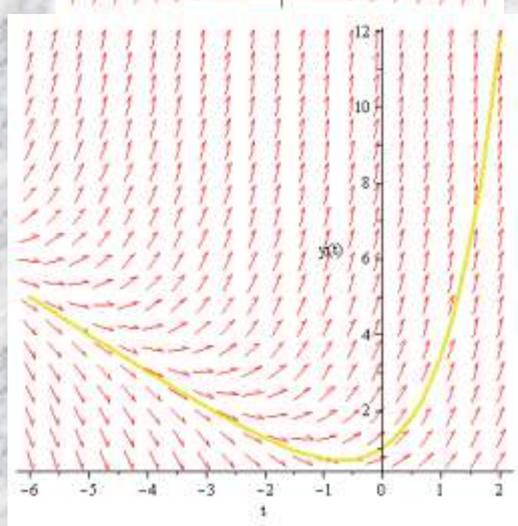

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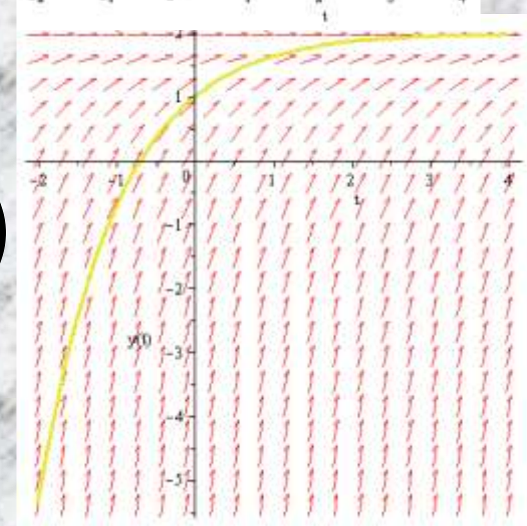

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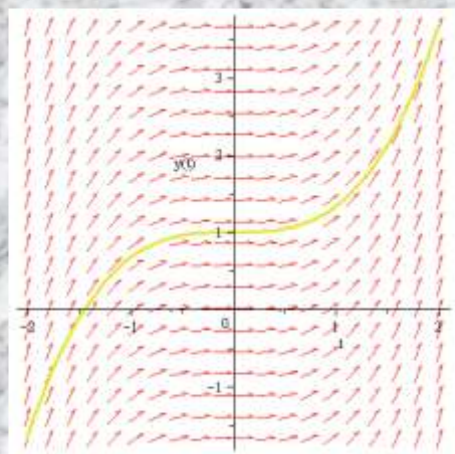

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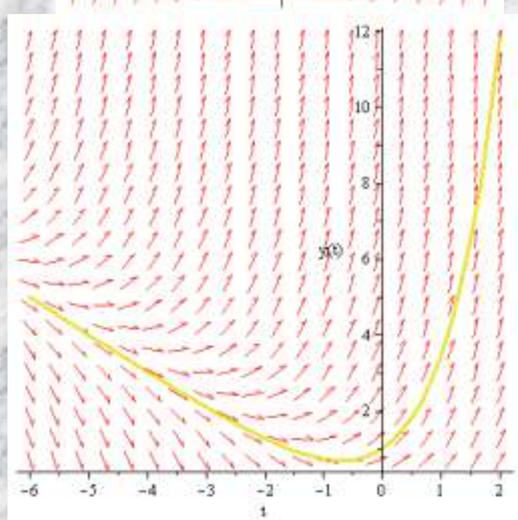

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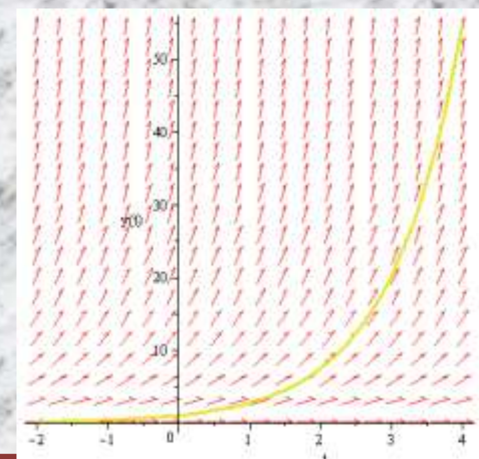

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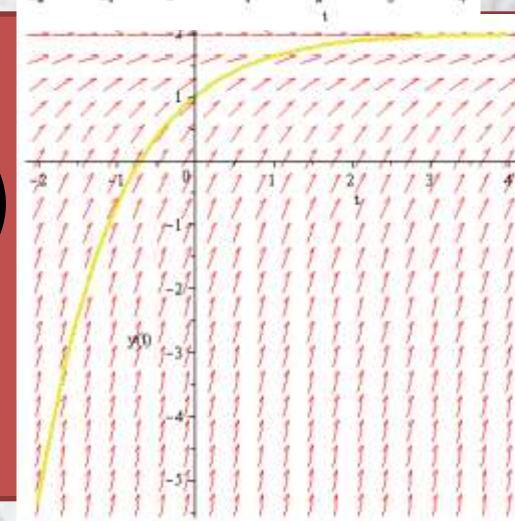

Autonomous Differential Equations

Suppose we have y' = F(y,x)

Notice if F(x,y) only depends on x then shifting the direction field up/down doesn't change it.

Notice if F(x,y) only depends on y then shifting the direction field left/right doesn't change it.

We call this **autonomous.**

Euler’s Method

Euler’s method is a way to approximate the solutions to a differential equation.

The idea is to start at the initial condition, move along the tangent line for a little bit, then adjust the direction and move along a new tangent line.

Euler's Method Continued

If $y' = F(x,y)$, the step distance is h, and the initial point is (x_0,y_0) then we have

$x_1 = x_0+h$ $\quad$ $y_1=y_0+hF(x_0,y_0)$

$x_2 = x_1+h$ $\quad$ $y_2=y_1+hF(x_1,y_1)$

$x_3 = x_2+h$ $\quad$ $y_3=y_2+hF(x_2,y_2)$

…

$x_{n+1} = x_n+h$ $\quad$ $y_{n+1}=y_n+hF(x_n,y_n)$

The smaller h is the better the approximation

Euler's Method Example

Let $y' = x+y$, $(0,1)$ is on the curve and $h = 1$

$x_1 = 1$ $\quad$ $y_1 = 0+1 = 1$

$x_2 = 2$ $\quad$ $y_2 = 1+1 = 2$

$x_3 = 3$ $\quad$ $y_3 = 2+2 = 4$

$x_4 = 4$ $\quad$ $y_4 = 3+4 = 7$

$x_5 = 5$ $\quad$ $y_5 = 4+7 = 11$

The smaller h is the better the approximation