Math 104-006

Chapter 12.8: Power Series

Outline For Today

- Power Series

Power Series

- A power series is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots$$

- Where x is a variable and the c_i are the coefficients of the series

Power Series Continued

- A power series may converge for some values of x and diverge for others.
- The sum of the series is a function

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \ldots$$

Whose domain is the collection of x for which the series converges.

Example

- Consider

$$f(x) = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots$$

We know this converges if and only if -1 < x < 1

Further in this case f(x) = 1/(1-x)

Example

- If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n}$ what is the domain of f(x)?

A) All x

B) -1 < x <1

C) -1 $\leq$ x < 1

D) -1 < x $\leq$ 1

E) -1 $\leq$ x $\leq$ 1

F) None of the above

Example

- If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n}$ what is the domain of f(x)?

A) All x

B) $-1 < x < 1$

C) $-1 \leq x < 1$

D) $-1 < x \leq 1$

E) $-1 \leq x \leq 1$

F) None of the above

Power Series in (x-a)

- A “power series in (x-a)” or “a power series centered at a” or “a power series about a” is of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \ldots$$

Radius of Convergence

- For any given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$
- There are only three options for the radius of convergence.

Radius of Convergence Continued

- Radius of Convergence is 0
- In this case the series $\sum_{n=0}^{\infty} c_n (x-a)^n$

converges when x=a and diverges everywhere else

Radius of Convergence Continued

- Radius of Convergence is a real number R
- In this case the series $\sum_{n=0}^{\infty} c_n (x-a)^n$

converges if $|x-a| < R$ and diverges if $|x-a| > R$

Radius of Convergence Continued

- Radius of Convergence is ∞
- In this case the series $\sum_{n=0}^{\infty} c_n (x-a)^n$

converges for all real x.

Interval of Convergence

- For any given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$
- The interval of convergence is the domain of the function.
- There are only six options for the interval of convergence.

Interval of Convergence: Radius is 0

- If the radius of Convergence of $\sum_{n=0}^{\infty} c_n (x-a)^n$ is 0.

- Then the interval of convergence is [a, a]

Interval of Convergence: Radius is ∞

- If the radius of Convergence of $\sum_{n=0}^{\infty} c_n (x-a)^n$ is ∞.

- Then the interval of convergence is (- ∞, ∞)

Interval of Convergence: Radius is R

- If the radius of Convergence of $\sum_{n=0}^{\infty} c_n (x-a)^n$ is R.

- Then there are four possible interval of convergences

  (-R + a, a +R)

  [-R + a, a +R)

  (-R + a, a +R]

  [-R + a, a +R]

Example

- What is the interval of convergence of $\sum_{n=0}^{\infty} n!\, x^n$?

A) (- 1, 1)

B) (- 1, 0)

C) (- 1, 0]

D) [0, 0]

E) (- ∞, ∞)

F) None of the above

Example

- What is the interval of convergence of $\sum_{n=0}^{\infty} n!\, x^n$?

A) (- 1, 1)

B) (- 1, 0)

C) (- 1, 0]

D) [0, 0]

E) (- ∞, ∞)

F) None of the above

Example

- What is the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?

A) (- 1, 1)

B) (- 1, 0)

C) (- 1, 0]

D) [0, 0]

E) (- ∞, ∞)

F) None of the above

Example

- What is the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?

A) (- 1, 1)

B) (- 1, 0)

C) (- 1, 0]

D) [0, 0]

E) $(-\infty, \infty)$

F) None of the above