Math 104-006

Chapter 12.7: Strategies for Testing Series

Outline For Today

- General Approach to Convergence and Divergence of Series.

General Method

- We now have several methods for determining whether or not a series converges.
- As with integration we want to classify a sereis with respect to its form to tell us which method we should use.

1) p-Series

- If the series is of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ then it is a p-series

Which is convergent if p>1 and divergent if p $\leq$ 1

2) Geometric Series

- If the series is of the form $\sum_{n=0}^{\infty} ar^n$ or $\sum_{n=0}^{\infty} ar^n$

then it is a geometric series which converges if |r| < 1 and diverges if |r| $\geq$ 1

- If it converges then $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

- Some preliminary manipulations may be needed to get the series in the correct form.

3) Comparison with p-series

- If the series has a form that is similar to a p-series or a geometric series then one of the comparison tests should be considered.
- In particular if an is a rational function or algebraic function of n (involving roots of polynomials), then the series should be compared with a p-series. The value of p should be chosen by only keeping the highest power of the numerator and denominator

3) Comparison With Absolute Value

- The comparison tests apply only to series with positive terms. But if $\sum_{n=0}^{\infty} a_n$ has negative terms we can use can use the comparison test with $\sum_{n=0}^{\infty} | a_n |$ to test for absolute convergence

4) Sequence Limit Isn't 0

- If you can see at a glance that $\lim_{n\to\infty} a_n \neq 0$

then $\sum_{n=0}^{\infty} a_n$ diverges

5) Alternating Series Test

- If the series is of the form

$$\sum_{n=0}^{\infty}(-1)^n b_n \text{ or } \sum_{n=0}^{\infty}(-1)^{n-1} b_n$$

Then the alternating series test is a possibility

6) Ratio Test

- If the series involves factorials or other products (including constants raised to the nth power) the ratio test is often useful.
- Remember though that

$$\left|\frac{a_{n+1}}{a_n}\right| \to \infty \quad \text{as} \quad n \to \infty$$

for all p-series and therefore for all algebraic functions of n. Thus the ration test should not be used for such series.

7) Root Test

- If $a_n = (b_n)^n$ then the root test might be useful.

8) Integral Test

- If $a_n = f(n)$ where

$$\int_1^\infty f(x)dx$$ is easily evaluated then the

integral test works (assuming the hypotheses of this test are satisfied.