Math 104-006

Chapter 12.6: Absolute Convergence and the Ratio and Root Tests

Outline For Today

- Absolute Convergence
- Conditional Convergence
- Ratio Test
- Root Test

Absolute Convergence

- An series is *Absolutely Convergent* if the series of absolute values converges.

- i.e. $\sum_{n=0}^{\infty} a_n$ is *absolutely convergent* if

$$\sum_{n=0}^{\infty} | a_n |$$ converges.

Conditionally Convergence

- An series is *Conditionally Convergent* if it is convergent but not absolutely convergent.

- i.e. $\sum_{n=0}^{\infty} a_n$ is *conditionally convergent* if

$\sum_{n=0}^{\infty} a_n$ converges and $\sum_{n=0}^{\infty} | a_n |$ diverges.

Theorem

- If $\sum_{n=0}^{\infty} a_n$ is absolutely convergent then

  $\sum_{n=0}^{\infty} a_n$ is convergent

Example

- Does $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converge?

A) Yes

B) No

C) Neither

Example

- Does $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converge?

A) Yes

B) No

C) Neither

Ratio Test

- If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$ then

(i) If L < 1 then the series $\sum_{n=0}^{\infty} a_n$ is absolutely convergent

(ii) If L > 1 then the series $\sum_{n=0}^{\infty} a_n$ is divergent

(iii) If L = 1 then the ration test is inconclusive.

Example

- Does $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ converge?

A) Yes

B) No

C) Neither

Example

- Does $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ converge?

A) Yes

B) No

C) Neither

Example

- Does $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge?

A) Yes

B) No

C) Neither

Example

- Does $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converge?

A) Yes

B) No

C) Neither

Root Test

- If $\lim_{n\to\infty} \sqrt[n]{a_n} = L$ then

(i) If L < 1 then the series $\sum_{n=0}^{\infty} a_n$ is absolutely convergent

(ii) If L > 1 then the series $\sum_{n=0}^{\infty} a_n$ is divergent

(iii) If L = 1 then the ration test is inconclusive.

Example

- Does $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^{n}$ converge?

A) Yes

B) No

C) Neither

Example

- Does $\sum_{n=1}^{\infty}\left(\frac{n+1}{2n-1}\right)^{n}$ converge?

A) Yes

B) No

C) Neither

Another Example

- Does $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converge?

A) Yes

B) No

C) Neither

Another Example

- Does $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converge?

A) Yes

B) No

C) Neither

Rearrangement of Terms

- If $\sum_{n=0}^{\infty} a_n$ is absolutely convergent and

$\sum_{n=0}^{\infty} b_n$ is a rearrangement of the terms then

$\sum_{n=0}^{\infty} b_n$ converges and $\sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} b_n$

Rearrangement of Terms

- If $\sum_{n=0}^{\infty} a_n$ is conditionally convergent then for every number L there is a rearrangement of the terms $\sum_{n=0}^{\infty} b_n$ such that $\sum_{n=0}^{\infty} b_n = L$ and another rearrangement $\sum_{n=0}^{\infty} c_n$ which diverges.