Math 104

3rd Midterm, Make-up Exam (Tuesday December 2, 2008)

Before solving the problems, please read carefully the instructions below and fill in the front page using capital block letters.

Full name: ______________________________

Student Number: ______________________________

Instructor: ______________________________

TA: ______________________________

Recitation Section: ______________________________

Corrector's signature	Sum of points	Grade

(1) Before starting to solve any problem, please fill in the front sheet.
(2) Please write your answers in the table "Your Answer" on the page of each problem. You can *only* get credit for a problem if an answer appears in this table.
(3) No calculators, computers or mobile phones are permitted.
(4) Only one sheet of letter size paper (8.5 in. by 11 in.) is permitted (written on both sides). No scratch paper is permitted.
(5) We are *not* responsible for providing extra paper. Please use the back of the sheets.
(6) Each problem is worth 10 points. We do *not* give partial credit. However, you may get *no* credit for a problem if no work is shown or if, in the sole opinion of the grader, your work (calculations) is incomplete or unclear or if your work (calculations) does not support your answer choice sufficiently.
Please show complete calculations and indicate clearly should you use the back of a sheet for your work.
(7) After the end of the exam you must stop writing and remain quietly seated at your place, your exam copy face-down on your desk, until all copies of the exam are collected.
(8) Exams must remain stapled.

Problem 1. Which statement is true for

$$(1)\ \sum_{n=1}^{\infty} \frac{2+3n}{3+2n}; \qquad (2)\ \sum_{n=6}^{\infty} \frac{1}{n\sqrt{n-5.5}}; \qquad (3)\ \sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$$

Answer choices:

(A) All three are convergent
(B) Only (1) and (3) are convergent
(C) Only (2) and (3) are convergent
(D) Only (1) is convergent
(E) Only (2) is convergent
(F) Only (3) is convergent

Your answer:	

Problem 2. Which statement is true for

$$(1)\ \sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}; \qquad (2)\ \sum_{n=1}^{\infty} ne^{-n^2}; \qquad (3)\ \sum_{n=1}^{\infty}\frac{3^n+n^4}{4^n+n^3}$$

Answer choices:

(A) All three are convergent
(B) Only (1) and (3) are convergent
(C) Only (2) and (3) are convergent
(D) Only (1) is convergent
(E) Only (2) is convergent
(F) Only (3) is convergent

Your answer:	

Problem 3. Decide if

$$\sum_{n=0}^{\infty} e^{-n}$$

converges or diverges. If it converges, calculate the sum.

Answer choices:

(A) The series diverges
(B) $\frac{1}{e-1}$
(C) $\frac{e}{e-1}$
(D) $\frac{2e}{e-1}$
(E) 1
(F) e

Your answer:	

Problem 4. Which statement is true for

$$(1)\ \sum_{n=1}^{\infty}\frac{(-4)^n}{3^{2n+1}}; \qquad (2)\ \sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\ln n}; \qquad (3)\ \sum_{n=1}^{\infty}(-1)^n\frac{3^{n+1}}{n!}$$

Answer choices:

(A) All three are convergent
(B) Only (1) and (3) are convergent
(C) Only (2) and (3) are convergent
(D) Only (1) is convergent
(E) Only (2) is convergent
(F) Only (3) is convergent

Your answer:	

Problem 5. Determine the radius of convergence for the following power series,

$$(1)\ \sum_{n=1}^{\infty} \frac{x^n}{(n+1)^3}; \qquad (2)\ \sum_{n=1}^{\infty} \frac{2^n x^n}{n!};$$

Answer choices:

(A) Both series have radius of convergence $R = 0$.
(B) Both series have radius of convergence $R = 1$.
(C) Both series have radius of convergence $R = \infty$.
(D) Series (1) has radius of convergence $R = 0$, while series (2) has $R = 1$
(E) Series (1) has radius of convergence $R = 0$, while series (2) has $R = \infty$
(F) Series (1) has radius of convergence $R = 1$, while series (2) has $R = \infty$

Your answer:	

Problem 6. Determine for each of the following series whether it is divergent, conditionally convergent, or absolutely convergent.

(1) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^2-1}{n^2+n^3}$; (2) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+\sqrt{n}}{1-\sqrt{n}}$; (3) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n^3)}{n^3}$

Answer choices:

(A) (1) conditionally convergent; (2) divergent; (3) absolutely convergent.
(B) (1), (2) and (3) conditionally convergent.
(C) (1) conditionally convergent; (2) and (3) absolutely convergent.
(D) (1) and (2) divergent; (3) conditionally convergent.
(E) (1) divergent; (2) conditionally convergent; (3) absolutely convergent.
(F) (1) divergent; (2) and (3) conditionally convergent.

Your answer:	

Problem 7. Find the MacLaurin series (Taylor series about $a = 0$) for

$$\frac{2x}{1+x^3}$$

Answer choices:

(A) $\sum_{n=0}^{\infty} 2x^{3n+1} = 2x + 2x^4 + 2x^7 + \ldots$
(B) $\sum_{n=0}^{\infty}(-1)^n \cdot 2x^{3n+1} = 2x^2 - 2x^4 + 2x^7 - \ldots$
(C) $\sum_{n=1}^{\infty} 2x^{3n} = 2x^3 + 2x^6 + 2x^9 + \ldots$
(D) $\sum_{n=0}^{\infty}(-1)^n \cdot 2x^{3n} = 2 - 2x^3 + 2x^6 - \ldots$
(E) $\sum_{n=0}^{\infty} \frac{2x^{3n+1}}{n!} = 2x + 2x^4 + \frac{2x^7}{2!} + \frac{2x^{10}}{3!} + \ldots$
(F) $\sum_{n=0}^{\infty}(-1)^n \cdot \frac{2x^{3n+1}}{n!} = 2x - 2x^4 + \frac{2x^7}{2!} - \frac{2x^{10}}{3!} + \ldots$

Your answer:	

Problem 8. Find the MacLaurin series (Taylor series about $a = 0$) for the function

$$F(x) = \int_0^x t \cdot \ln(1+t)\, dt.$$

Answer choices:

(A) $\sum_{n=3}^{\infty} \frac{x^n}{n(n-2)} = \frac{1}{3}x^3 + \frac{1}{2\cdot 4}x^4 + \frac{1}{3\cdot 5}x^5 + \dots$

(B) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{x^n}{n(n-2)} = \frac{1}{3}x^3 - \frac{1}{2\cdot 4}x^4 + \frac{1}{3\cdot 5}x^5 - \dots$

(C) $\sum_{n=3}^{\infty} \frac{x^n}{n-2} = x^3 + \frac{1}{2}x^4 + \frac{1}{3}x^5 + \dots$

(D) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{x^n}{n-2} = x^3 - \frac{1}{2}x^4 + \frac{1}{3}x^5 - \dots$

(E) $\sum_{n=3}^{\infty} \frac{x^n}{(n-2)!} = x^3 + \frac{1}{2!}x^4 + \frac{1}{3!}x^5 + \dots$

(F) $\sum_{n=3}^{\infty} (-1)^{n+1} \frac{x^n}{(n-2)!} = x^3 - \frac{1}{2!}x^4 + \frac{1}{3!}x^5 - \dots$

Your answer:	

Problem 9. Use the degree 4 MacLaurin polynomial

$$f(x) \approx c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$$

for the function $f(x) = \ln(1 + x)$ to find an approximate value for $\ln(2)$.

Answer choices:

(A) 5/12
(B) 1/2
(C) 7/12
(D) 2/3
(E) 3/4
(F) 5/6

Your answer:	

Problem 10. What is the interval of convergence of the following series?

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-2)^n}{n\cdot 2^n}$$

Answer choices:

(A) $(1/2, 7/2)$
(B) $[1/2, 7/2)$
(C) $(1/2, 7/2]$
(D) $(0, 4)$
(E) $[0, 4)$
(F) $(0, 4]$

Your answer:	