Math 104

2nd Midterm with Solutions (Nov. 4th 2008, 8:05pm – 8:55pm)

Problem 1. Evaluate $\int_1^2 \frac{1}{x^2(x+1)}dx.$ **Answer:** $\ln\left(\frac{3}{4}\right)+\frac{1}{2}$

Problem 2. Determine $\int_0^1 x^3\sqrt{1+x^2}dx.$ **Answer:** $\frac{2}{15}(1+\sqrt{2})$

Problem 3. Approximate $\int_0^4 x^2dx$ using the trapezoidal rule with $n=4$ subdivisions.

Answer: 22

Problem 4. Evaluate $\int_0^\infty 2xe^{-x^2+1}dx.$ **Answer:** e

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Problem 5. Evaluate $\int_1^5 \frac{1}{\sqrt{x-1}}dx.$ **Answer:** 4

Problem 6. Determine the arc length of the graph of $f(x)=\frac{1}{2}(e^x+e^{-x})$ from $x=0$ to $x=\ln(2)$. **Answer:** 3/4

Problem 7. Determine the arc length of curve of $y=\frac{x^3}{12}+\frac{1}{x}$ from $x=1$ to $x=2$. **Answer:** 13/12

Problem 8. Determine the area of the surface of revolution obtained by rotating the curve $y=\frac{1}{3}x^3$ between $x=0$ and $x=\sqrt[4]{3}$ around the x-axis. **Answer:** $\frac{7}{9}\pi$

Problem 9. Which of the following statements is true given the sequences:

(1) $\left\{\frac{n+\sqrt{n}}{e^n+1}\right\}_{n=1}^\infty$; (2) $\left\{\frac{2^{n+1}}{3^n}\right\}_{n=1}^\infty$; (3) $\left\{(-1)^n\frac{3^n}{2^{n+1}}\right\}_{n=1}^\infty$.

Answer: Only the sequences (1) and (2) are convergent.

Problem 10. Which statement is true about the sequence $a_n=(-1)^n\frac{\sin(n^2)}{n}$?
Answer: The sequence is convergent, which can be shown using the squeeze (sandwich) theorem.

Problems may have appeared in a different order on you exam.
No guarantee for the correctness of the answers.