

## (Math 170) Homework 9:

Due March 30, 2007

**Definition 0.0.1.** Let  $G(m, n)$  be the function defined as follows. If  $F(m)$  is the function  $f(x_1, \dots, x_l)$  and  $n = 2^{k_0} \cdot 3^{k_1} \cdots \text{Prime}(r)^{k_r}$  then

- $G(m, n) = 0$  if  $f(k_0, \dots, k_l)$  is undefined
- $G(m, n) = 1$  if  $f(k_0, \dots, k_l)$  is defined

**Definition 0.0.2.** Let  $\mathcal{H} = \{\text{Functions obtained from the recursive functions and } G \text{ by the recursive operations (i.e. Composition, Primitive Recursion, and the } \mu \text{ operator)}.\}$

Exercise 1: Describe a method for coding a function in  $\mathcal{H}$  by a natural number. I.e. Find a function  $H$  from the natural numbers to  $\mathcal{H}$  such that for every function  $h \in \mathcal{H}$  there is at least one natural number  $n$  such that  $H(n) = h$ .

Your method should agree with the method described in class on all natural numbers such that  $2^6$  does not divide  $n$ .

Exercise 2: Let  $J(m, n)$  be the function defined as follows. If  $H(m)$  is the function  $h(x_1, \dots, x_l)$  and  $n = 2^{k_0} \cdot 3^{k_1} \cdots \text{Prime}(r)^{k_r}$  then

- $J(m, n) = 0$  if  $f(k_0, \dots, k_l)$  is undefined
- $J(m, n) = 1$  if  $f(k_0, \dots, k_l)$  is defined

(Note here  $H$  is the function defined in the previous exercise).

Is  $J \in \mathcal{H}$ ? Prove your answer. (Hint: Use a diagonal argument)

Exercise 3: Give an example of a partial (or total) function which is recursive but not primitive recursive. Justify your answer.

Exercise 4: Let  $m(x, y) = x \cdot y$ . Find a definition of  $m$  in terms of the basic primitive recursive operations and functions.

Exercise 5: Which of the following sets are recursive? Which are recursively enumerable? Which are neither? You may assume the Church-Turing Thesis.

- (a)  $\{2^a \cdot 3^b \cdot 5^c \cdot 7^d : a = b! + c \cdot d\}$
- (b)  $\{2^n \cdot 3^m : \text{where } n \text{ is the } m\text{th digit of } \pi\}$ .
- (c)  $\{2^n \cdot 3^m : \text{where } F(n) \text{ and } F(m) \text{ are different total recursive functions}\}$