

Homework 12 Math 170: Ideas in  
Mathematics (Spring 2007)

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Due April 20, 2007

- (1) Suppose you are at a cafeteria which has the following
- 4 pieces of fruit, one of which is a pineapple.
  - 3 main courses, one of which is a piece of pizza
  - 4 deserts, two of which are pieces of pound cake.

Assuming nothing else in the cafeteria starts with the letter “P” and you choose randomly choose one piece of fruit, one main course and one desert, what is the probability that everything you choose will start with the letter “P”?

- (2) Consider the discrete dynamical system given by  $P_{n+1} = \frac{1}{2}(P_n^2 + P_n)$ . What are the equilibrium and are they stable or unstable from the left and right? (The attached graph may be useful)
- (3) Consider the function  $f(x) = x^4 - 4x^3 + 9x^2 - 4x + 8$ . If we know that  $f(2 - 2i) = 0$  and  $f(-i) = 0$  find all complex numbers  $a + bi$  such that  $f(a + bi) = 0$  and find their norms.
- (4) Suppose there are 20 dogs in a kennel and suppose 9 of them are male and 11 of them are female. Further suppose that 7 of the dogs are German Shepards and that 4 of the German Shepards are female. IF a dog is chosen at random from those in the kennel, what is the probability that it will satisfy at least one of the following two conditions
- It is female
  - It is a German Shepard

- (5) Let  $a, b, c, d$  be integers. Suppose  $(a, b) \sim (c, d)$  if  $a - c = 2n$  and  $b - d = 6m$  for some integers  $n, m$ . Then  $\sim$  is an equivalence relation on  $\mathbb{Z} \times \mathbb{Z} = \{(a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}\}$ . How many elements does  $\mathbb{Z} \times \mathbb{Z} / \sim$  have? (i.e. how many equivalence classes are there under the equivalence relation  $\sim$ ?)

Recall that the first 3 rows of Pascal's triangle are

Row 0		1	
Row 1	1	1	
Row 2	1	2	1

With the first 1 being the 0th element.

- (6) What is the 5th element of the 8th row of Pascal's Triangle?
- (7) If there are 3 people in a room what is the probability that no two have the same birthday.
- (8) What does  $[2, 5, 2, 5, \dots]$  equal (express your answer as a fraction with square roots)?
- Let  $a = 792, b = 847$
- (9) What is the greatest common divisor of  $a$  and  $b$ ?
- (10) Find integers  $x, y$  such that  $ax + by = \gcd(a, b)$ .

- (11) A jar contains 5 yellow marbles, 3 blue marbles, and 1 red marble. If you randomly choose two marbles from the jar (replacing the first before you choose the second) what is the probability that the two marbles will be the same color?
- (12) What is the coefficient of  $x^4$  in  $(2 + 2x)^7$ ?
- (13) Notice that  $616 = 11 \cdot 7 \cdot 2^3$ . How many numbers less than 616 have inverses in  $\mathbb{Z}/616$ ?
- (14) Suppose that every ticket in the National lottery has a 1 in a 15 chance of winning. Suppose that you buy a single ticket every day for 8 days. What is the probability that you will loose every time?

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
1	2	3	4	5	6	7	8	9	10	11	12	13
<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
14	15	16	17	18	19	20	21	22	23	24	25	26

- (15) Use a Vigenere cipher with keyword FUN to encode the message  
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Let  $PlusThree(n) = n + 3$  Define  $h(n)$  by primitive recursion with

- Base Function:  $h(0) = 0$
- Inductive Function:  $h(n + 1) = PlusThree( \pi_1(h(n), n) )$

Here  $\pi_1$  is the function which projects onto the first coordinate.

- (16) What is  $h(2)$ ,  $h(3)$  and  $h(4)$ ?
- (17) Find a number  $a$  such that  $a^5 = 7 \pmod{9}$ .
- (18) Let  $f(x, y) = (-2x + 1)y + x^2$ . What is  $\mu f(x, 1)$ ? What is  $\mu f(1, y)$ ?