(1a) If there are 5 people in a room what is the probability that at least two were born in the same hour of the day (e.g. 2 were born between 3 am - 4 am).

(1b) If there are 5 people in a room what is the probability that at least two of them share a birthday?

(1c) If there are 4 people in a room, what is the probability that no two were born on the same day of the week?

(2a) Suppose you have the following pieces of clothing in your closet.

- 3 pairs of pants, 1 of which is blue.
- 5 pairs of socks, 2 pairs of which are blue.
- 4 shirts, 2 of which are blue.

If in the morning you randomly put on 1 pair of pants, 1 shirt and 1 pair of socks what is the probability that everything you put on is blue (note socks are in pairs which can't be separated).

(2b) Suppose in your library you have the following.

- 4 books on math, 3 of which have yellow covers.
- 6 books on philosophy, 1 of which has a yellow cover
- 5 books on travel, 3 of which have a yellow cover.

If you were to randomly select one book on math, one book on philosophy, one book on travel from your library. What is the probability that all three would have yellow covers?

(2c) Suppose you have 3 piles of cards.

- In the first pile there are 7 cards, 3 of which are clubs
- In the second pile there are 4 cards, 1 of which is a clubs
- In the third pile there are 8 cards, 2 of which are clubs

If you were to randomly choose a card from each pile, what is the probability that all three would be clubs?

(3a) Suppose 100 people are in a room, 50 of which are male and 50 of which are female. Suppose 20 of the people are from Pennsylvania, and 12 of the people are men from Philadelphia. Suppose a random person is chosen from those in the room. What is the probability that the person chosen will either

- Be a man.

OR

- They will be from Pennsylvania.

(3b) Suppose there are 100 balloons in a room. Suppose 50 of the balloons are red, and 50 are blue. Further suppose 65 of the balloons are filled with helium and that there are 25 red balloons filled with helium. Suppose a random balloon is chosen from those in the room. What is the probability that the balloon chosen either will

- Be red.

OR

– Be filled with helium.

(3c) There are 168 primes less than 1000, and 250 numbers less 1000 which have a remainder of 1 when divided by 4. Further, 79 of the primes less than 1000 have a remainder of 1 when divided by 4. If you were to randomly select a number less than 1000 what is the probability that either

– It is a prime.

OR

– The remainder when divided by 4 is 1.

(4a) Consider the function $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 5$. If we know that $f(2+i) = 0$ and $f(i) = 0$ find all complex numbers $a + bi$ such that $f(a + bi) = 0$ and find their norms.

(4b) Consider the function $f(x) = x^3 - 4x^2 + 6x - 4$. If we know that $f(2) = 0$ and $f(1+i) = 0$ find all complex numbers $a + bi$ such that $f(a + bi) = 0$ and find their norms.

(4c) Consider the function $f(x) = x^4 - 2x^3 + 9x^2 - 8x + 20$. If we know that $f(1+2i) = 0$ and $f(2i) = 0$ find all complex numbers $a + bi$ such that $f(a + bi) = 0$ and find their norms.

(5a) What does $[1, 3, 1, 3, \ldots]$ equal?

(5b) What does $[2, 1, 2, 1, \ldots]$ equal?

(5c) What does $[2, 3, 2, 3, \ldots]$ equal?

(6a) Let a, b, c, d be integers. Suppose $(a, b) \sim (c, d)$ if $a - c = 2n$ and $b-d = 5m$ for some integers n, m. Then $\sim$ is an equivalence relation on $\mathbb{Z}\times\mathbb{Z} = \{(a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}$. How many elements does $\mathbb{Z}\times\mathbb{Z}/\sim$ have? (i.e. how many equivalence classes are there under the equivalence relation $\sim$?)

(6b) Let a, b, c, d be integers. Suppose $(a, b) \sim (c, d)$ if $a - c = 4n$ and $b-d = 3m$ for some integers n, m. Then $\sim$ is an equivalence relation on $\mathbb{Z}\times\mathbb{Z} = \{(a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}$. How many elements does $\mathbb{Z}\times\mathbb{Z}/\sim$ have? (i.e. how many equivalence classes are there under the equivalence relation $\sim$?)

(6c) Let a, b, c, d be integers. Suppose $(a, b) \sim (c, d)$ if $a - c = 3n$ and $b-d = 3m$ for some integers n, m. Then $\sim$ is an equivalence relation on $\mathbb{Z}\times\mathbb{Z} = \{(a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}$. How many elements does $\mathbb{Z}\times\mathbb{Z}/\sim$ have? (i.e. how many equivalence classes are there under the equivalence relation $\sim$?)

(7a) What is the coefficient of x^4 in $(1 + 3x)^6$?

(7b) What is the coefficient of x^3 in $(1 + 4x)^6$?

(7c) What is the coefficient of x^3 in $(2 + 3x)^6$?

Let $a = 625$, $b = 1120$

(8a) What is the greatest common divisor of a and b?

(9a) Find integers x, y such that $ax + by = gcd(a, b)$. Let $c = 1323$, $d = 375$

(8b) What is the greatest common divisor of c and d?

(9b) Find integers x, y such that $cx + dy = gcd(c, d)$. Let $e = 780$, $f = 924$

(8c) What is the greatest common divisor of e and f?

(9c) Find x, y such that $ex + fy = gcd(e, f)$.

(10a) Notice that $198 = 2 \cdot 3^2 \cdot 11$. How many numbers less than 198 have inverses in $\mathbb{Z}/198$?

(10b) Notice that $325 = 5^2 \cdot 13$. How many numbers less than 325 have inverses in $\mathbb{Z}/325$?

(10c) Notice that $360 = 2^3 \cdot 3^2 \cdot 5$. How many numbers less than 360 are relatively prime to 360?

(11a) Find a number a such that $a^5 = 5 \mod 14$.

(11b) Find a number a such that $a^7 = 6 \mod 11$.

(11c) Find a number a such that $a^7 = 7 \mod 10$.

(12a) Use a Vigenere cipher with keyword HOME to encode the message

EXAMS ARE FUN

(12b) Use a Vigenere cipher with keyword ALMOST to encode the message

EXAMS ARE FUN

(12c) Use a Vigenere cipher with keyword MATH to encode the message

EXAMS ARE FUN

(13a) Let $TimesThree(n) = 3n$ Define $h(n)$ by primitive recursion with

- Base Function: $h(0) = 1$
- Inductive Function: $h(n + 1) = TimesThree(\ \pi_1(h(n), n)\ )$

Here π_1 is the function which projects onto the first coordinate. What is the pair of numbers $(h(2), h(3))$?

(13b) Let $PlusTwo(n) = n + 2$ Define $h(n)$ by primitive recursion with

- Base Function: $h(0) = 2$
- Inductive Function: $h(n + 1) = PlusTwo(\ \pi_1(h(n), n)\ )$

Here π_1 is the function which projects onto the first coordinate. What is the pair of numbers $(h(2), h(3))$?

(13c) Let $TimesTwo(n) = 2n$ Define $h(n)$ by primitive recursion with

- Base Function: $h(0) = 1$
- Inductive Function: $h(n + 1) = TimesTwo(\ \pi_1(h(n), n)\ )$

Here π_1 is the function which projects onto the first coordinate. What is the pair of numbers $(h(2), h(3))$?

(14a) Let $f(x, y) = (x - 3)^2 + (y - 2)^2$. What is $(\mu f(3, y), \mu f(x, 2))$?

(14b) Let $f(x, y) = y^3 - xy$. What is $\mu f(4, y)$? What is $\mu f(x, 2))$?

(14c) Let $f(x, y) = y^2x - x^2$. What is $\mu f(4, y)$? What is $\mu f(x, 2))$?