Math 170 Final Formula Sheet

Probability

Formula 1

Suppose we have an Events A such that

- The Probability that Event A has Outcome X_1 is P_1
- The Probability that Event A has Outcome X_2 is P_2
- The Probability that (Event A has Outcome X_1) AND (Event A has Outcome X_2) is P_3

then

- The Probability that (Event A_1 has Outcome X_1) OR (Event A_2 has Outcome X_2) $= P_1 + P_2 - P_3$

Independent Events

Suppose we have two Events B_1, B_2

- Probability that Event B_1 has Outcome Y_1 is Q_1
- Probability that Event B_2 has Outcome Y_2 is Q_2

If Event B_1 and Event B_2 are independent (i.e. the out come of Event B_1 does not effect the outcome of Event B_2) then

- The Probability that (Event B_1 has Outcome Y_1) AND (Event B_2 has Outcome Y_2) $= Q_1 \times Q_2$.

Dynamical Systems

- Let $P_{n+1} = f(P_n)$ be a discrete dynamical system. Then the equilibrium of the system occur at values of P such that $P = f(P)$.

- An equilibrium P is

  - Stable from the left if and only if for all small enough ϵ, when $P_0 = P - \epsilon$ then $\lim_{n\to\infty} P_n = P$

  - Stable from the right if and only if for all small enough ϵ, when $P_0 = P + \epsilon$ then $\lim_{n\to\infty} P_n = P$

Complex Numbers and Polynomials

- Let $f(x)$ be a polynomial in x with real coefficients. Then $\overline{f(x)} = f(\overline{x})$. So, if $f(x) = 0$ then $f(\overline{x}) = 0$

- If $f(x) = ax^2 + bx + c$ where a, b, c are real numbers then $f(x) = 0$ if and only if
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $f(x)$ is a polynomial of degree n then f factors into n linear factors over the complex numbers.

- If a, b are real numbers then

  - $\overline{a + bi} = a - bi$

  - $\text{Norm}(a + bi) = \sqrt{a^2 + b^2}$

Pascal's Triangle and Binomial Formula

- Recall that the first 3 rows of Pascal's triangle are

Row 0			1		
Row 1		1		1	
Row 2	1		2		1

  With the first 1 being the 0th element.

- The mth element of the nth row is $\begin{pmatrix} n \\ m \end{pmatrix}$

- $\begin{pmatrix} n \\ m \end{pmatrix} = \dfrac{n!}{m!(n-m)!}$

- $\begin{pmatrix} n \\ m \end{pmatrix} = \begin{pmatrix} n-1 \\ m-1 \end{pmatrix} + \begin{pmatrix} n-1 \\ m \end{pmatrix}$ if $n \geq m$

- $(X+Y)^n = \sum_{i=0}^{n} \begin{pmatrix} n \\ i \end{pmatrix} X^i Y^{n-i}$

Euler's Formula and ϕ

- Let $p_1, p_2, \ldots, p_m$ be the prime factors of n. Then

$$\phi(n) = n \times \frac{p_1 - 1}{p_1} \times \frac{p_1 - 1}{p_1} \times \cdots \times \frac{p_m - 1}{p_m}$$

- There are $\phi(n)$ many numbers which have multiplicative inverses mod n and are less than n.

- There are $\phi(n)$ many numbers which are relatively prime to n and are less than n.

- Euler's Formula If $gcd(a, n) = 1$ then $a^{\phi(n)} = 1 \mod n$.

- If $gcd(a, n) = 1$ and $gcd(x, \phi(n)) = 1$ then there is a number m such that $(a^m)^x = a \mod n$. This m is such that $mx = 1 \mod \phi(n)$ (i.e such that there is a t with $mx = t\phi(n) + 1$)

Miscellaneous

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

- If $f(x)$ is a function, then $\mu f(x) = n$ is the least natural number (i.e. $\{0, 1, \dots\}$) such that $f(n) = 0$.

- Let X be a shape. Let X_n be the shape X scaled by a linear factor of $1/n$ (i.e. if X is a square with a side of length 1, then X_2 is a square with a side of length 1/2). If we can reconstruct X using m copies of X_n then the dimension of X is the number d such that

$$n^d = m$$

In other words the dimension of X is $\ln(m)/\ln(n) = \log_{10}(m)/\log_{10}(n)$.