

Questions From Math 170: Ideas in
Mathematics (Spring 2007)

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April 15, 2007

- (1) If there are 3 people in a room what is the probability that at least two were born on the same day of the week?
- (2) Suppose there are three rooms.
- The first room has 8 people, 3 of whose last name is Smith.
 - The second room has 6 people, 1 of whose last name is Smith.
 - The third room has 5 people, 2 of whose last name is Smith.

If one person is randomly chosen from each room, what is the probability that all three have last name Smith?

- (3) Suppose I have an 8 sided dice. If I roll this dice 4 times what are the odds that I will roll a 3 at least once?
- (4) Suppose a used car lot has 100 cars 20 of which are yellow and 80 of which are blue. Further suppose there are 10 cars in this lot which are made by Ford and there are 2 cars which are yellow Fords. If you randomly select a car from this lot what is the probability of it satisfying at least one of the following two conditions
- It is yellow.

– It is a Ford.

- (5) A jar contains 2 red marbles, 3 blue marbles, and 5 yellow marbles. If you randomly choose two marbles from the jar (replacing the first before you choose the second) what is the probability (to 3 decimal places) that the two marbles will be the same color?

- (6) Consider the discrete dynamical system given by $P_{n+1} = P_n^2 - 3P_n$. What are the equilibrium and are they stable or unstable from the left and right?

GIVE GRAPH OFF $y = x$ AND $y = x^2 - 3x$.

- (7) Consider the function $f(x) = x^4 - 6x^3 + 19x^2 - 54x + 90$. If we know that $f(-3i) = 0$ and $f(3 + i) = 0$ find all complex numbers $a + bi$ such that $f(a + bi) = 0$ and find their norms.

- (8) What does $[4, 5, 4, 5, \dots]$ equal to 4 decimal places?

- (9) Let a, b, c, d be integers. Suppose $a + bi \sim c + di$ if $a - c = 5n$ and $b - d = 6m$ for some integers n, m . Then \sim is an equivalence relation on the Gaussian integers (i.e. $\{a + bi : a, b \in \mathbb{Z}\}$). How many elements

does $\{a + bi : a, b \in \mathbb{Z}\} / \sim$ have? (i.e. how many equivalence classes are there under the equivalence relation \sim ?)

Recall that the first 3 rows of Pascal's triangle are

Row 0			1
Row 1	1		1
Row 2	1	2	1

With the first 1 being the 0th element.

(10) What is the 6th element of the 10th row of Pascal's Triangle?

(11) What is the coefficient of x^4 in $(4 + 5x)^8$?

Let $a = 5929$, $b = 1547$

(12) What is the greatest common divisor of a and b ?

(13) Find x, y such that $ax + by = \gcd(a, b)$.

(14) Notice that $750 = 2 \cdot 3 \cdot 5^2$. How many numbers less than 750 have inverses in $\mathbb{Z}/750$?

(15) Find a number a such that $a^7 = 2 \pmod{15}$.

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>
1	2	3	4	5	6	7	8	9	10	11	12	13
<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
14	15	16	17	18	19	20	21	22	23	24	25	26

(16) Use a Vigenere cipher with keyword CLASS to encode the message
CLASS IS ALMOST DONE

Let $Function(n) = 3n + 2$ Define $h(n)$ by primitive recursion with

- Base Function: $h(0) = 0$
- Inductive Function: $h(n + 1) = Function(\pi_1(h(n), n))$

Here π_1 is the function which projects onto the first coordinate.

(17) What is $h(2), h(3), h(4)$?

(18) Let $f(x, y) = x^2(y - 2)^2$. What is $(\mu f(3, y), \mu f(x, 1))$?