Definition 0.1. $\varphi : G \to G'$ is a homomorphism if $(\forall a, b)\varphi(a \circ b) = \varphi(a) \circ \varphi(b)$

$ker(\varphi) = \{g \in G : \varphi(g) = e\}$.

The Kernel is a subgroup of G

Definition 0.2. N is a normal subgroup of G if $(\forall g) gNg^{-1} = N$.

Theorem 0.3. *If $\varphi : G \to G'$ then $ker(\varphi)$ is a normal subgroup of G.*

Theorem 0.4. *If N is a normal subgroup of G then the map $\varphi(g) = gN$ is a group homomorphism with $ker(\varphi) = N$.*

Theorem 0.5. *If $\varphi : G \to G'$ is a homomorphism with $ker(\varphi) = N$ then $\varphi(a) = \varphi(b) \Leftrightarrow a = bn$ for some $n \in ker(\varphi) \Leftrightarrow a^{-1}b \in N$*

Theorem 0.6. *If $\varphi : G \to G'$ is a homomorphism then φ is injective if and only if $ker(\varphi) = \{e\}$*

Theorem 0.7. *If K, H is are subgroup of G then $K \cap H$ is a subgroup of G. Further if H is normal in G then $K \cap H$ is normal in K.*

Theorem 0.8. *Let $\varphi : G \to G'$ be a group homomorphism and let H' be a subgroup of G'. Define $\varphi^{-1}(H') = \{x \in G : \varphi(x) \in H\} = \overline{H}$*

(a) $\overline{H}$ is a subgroup of G

(a) If H' is a normal subgroup of G' then $\overline{H}$ is a normal subgroup of G

(a) $\overline{H}$ contains the $ker(\varphi)$

(a) φ restricts to a homomorphism $\overline{H} \to H'$

Definition 0.9. Let G, G' be groups. Then define $G \times G' = \{(g, g') : g \in G, g' \in G'\}$ with $(g, g') \circ (h, h') = (g \circ g', h \circ h')$

Theorem 0.10. *Maps $\Phi : H \to G \times G'$ are in a bijective correspondence with (φ, φ'), $\varphi : H \to G, \varphi' : H \to G'$ with $ker(\Phi) = ker(\varphi) \cap ker(\varphi')$.*

Definition 0.11. Let $A, B \subseteq G$. Then define $AB = \{ab : a \in A, b \in B\}$

Theorem 0.12. *Let $H, K \subseteq G$ are subgroups.*

(a) *If* $H \cap K = \{e\}$ *and* $p : H \times K \to G$ *with* $p(h,k) = hk$ *then* p *is injective and its image is* HK

(b) *If either* H *or* K *is normal then* $HK = KH$ *is a subgroup*

(c) *F* H, K *are both normal and* $H \cap K = \{e\}$ *and* $HK = G$ *then* $G \cong H \times K$.

Theorem 0.13 (1st Isomorphism Theorem)**.** *If* $\varphi : G \to G'$ *is surjective then* $G' \cong G/ker(\varphi)$