# Math 170 Final Formula Sheet

December 6, 2007

# 1 Mathematical Models

Let $M_{n+1} = f(M_n)$ be a mathematical model (with out the starting value specified).

**Definition 1.0.1.** $M$ is a *equilibrium* if $M = f(M)$. An equilibrium $M$ is a *stable equilibrium* if, whenever we let $M_0 = M \pm \epsilon$ then the limit of $M_n$ as $n \to \infty$ is $M$. Otherwise $M$ is unstable.

# 2 Mandelbrot and Julia Sets

**Definition 2.0.2.** The *Mandelbrot set* is the collection of complex numbers $c$ such that the mathematical model

$$M^c_{n+1} = (M^c_n)^2 + c, \qquad M_0 = 0$$

does not go to infinity as $n$ goes to infinity.

**Definition 2.0.3.** The *Julia set at $d$* is the collection of complex numbers $c$ such that the mathematical model

$$J^d_{n+1} = (J^d_n)^2 + d, \qquad J_0 = c$$

does not go to infinity as $n$ goes to infinity.

# 3 Complex Numbers

**Definition 3.0.4.** An *imaginary number* is one of the form $ai$ where $a$ is real and $i = \sqrt{-1}$

**Definition 3.0.5.** A *complex number* is one of the form $a+bi$ where $a, b$ are real and $i = \sqrt{-1}$

**Theorem 3.0.6.** *For all complex numbers,* $(a+bi)+(c+di) = (a+c)+(b+d)i$

**Theorem 3.0.7.** *For all complex numbers,* $(a+bi) \times (c+di) = (ac-bd) + (ad+bc)i$

**Theorem 3.0.8.** *Let* $(a+bi)$ *be a complex number and let* $r$ *is the distance of the point form 0 in the complex plane and let* $\theta$ *be the angle between the ray from 0 to* $a+bi$*) and (x-axis). Then*

- $a = r\cos(\theta)$
- $b = r\sin(\theta)$
- $\tan(\theta) = b/a$
- $r^2 = a^2 + b^2$

**Theorem 3.0.9** (Fundamental Theorem of Algebra)**.** *For every polynomial* $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ *with each* $a_i$ *a complex number, there is a complex number* $\alpha$ *such that* $f(\alpha) = 0$

**Theorem 3.0.10** (Quadratic Equation)**.** *If* $ax^2 + bx + c = 0$ *then*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# 4 Inverse Functions

**Definition 4.0.11.** Let $f(x), g(y)$ be functions. Then $f$ and $g$ are inverses if and only if

$$f(g(y)) = y \text{ for all } y$$

and

$$g(f(x)) = x \text{ for all } x$$

# 5 Logarithms and Exponents

**Theorem 5.0.12.**

$b^{\log_b(x)} = x$

$\log_b(b^x) = x$

$\log_b(a) = \log_k(a)/\log_k(b)$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^d) = d * \log_b(x)$

$\log_b(\sqrt[d]{x}) = \frac{\log_b(x)}{d}$

**Theorem 5.0.13.**

$b^{x+y} = b^x \times b^y$

$b^{x-y} = b^x/b^y$

$(b^x)^y = b^{xy}$

# 6 Dimension

**Definition 6.0.14.** Let $X$ be a self similar structure. If there is number $S$ such that we can shrink $X$ down by a factor of $X$ to get a shape $X'$ and then can piece $c$ copies of $X'$ together to get back our original shape $X$ then

$$S^d = c \text{ or } d = \log_S(c)$$

# 7 Infinite Series

**Theorem 7.0.15.** *$a+ar+ar^2+ar^3+\cdots$ is converges (i.e. equals a number) if and only if $|r| < 1$*

**Theorem 7.0.16.** *If $a + ar + ar^2 + ar^3 + \cdots$ converges then*

$$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$$

# 8 Change of Base

**Definition 8.0.17.** If $a_n, \ldots, a_0, a_{-1}, \ldots, a_{-k}$ are greater than or equal to 0 and less than $m$ then $a_n a_{n-1} \ldots a_0$ base $m = a_n * m^n + a_n * m^{n-1} + \cdots a_1 * m + a_0 + a_{-1}m^{-1} + \cdots + a_{-k}m^{-k}$