Practice Midterm (Math 371):

Fall 2006

Problem 1: Show that $c : G \times G \to G$ given by $c(g, x) = xg^{-1}$ is group action of G on itself.

Problem 2: Let $(A, +, 0)$ be a finitely generated Abelian group with no torsion elements. Show that the map $\varphi_d(x) : A \to A$ which is defined by $\varphi(x) = dx$ is an injective homomorphism for each $d \in \{1, 2, \dots\}$

Problem 3: Let $\circ : G \times X \to X$ be a group action. Show that $H = \{g \in G : (\forall s \in X) g \circ s = s\}$ is a normal subgroup of G.

Problem 4: Let N be a normal subgroup of G such that N is also a subset of a Sylow p-subgroup H of G. Show that N is a subset of every Sylow p-Group of G.

Problem 5: Find the orthonormal basis for $\mathbb{R}^3$ gotten by applying the Gram-Schmidt procedure to the basis vectors

$$v_1 = [1, 2, 2]$$

$$v_2 = [1, 4, 0]$$

$$v_3 = [9, 0, 0]$$

Problem 6: Let $\langle , \rangle$ be a Hermitian form on a complex vector space V. Let $\{v, w\}$ denote the real part of $\langle v, w \rangle$. Prove that if we consider V as a real vector space then $\{v, w\}$ is a symmetric bilinear form and if $\langle , \rangle$ is positive definite then $\{ , \}$ is positive definite.

Problem 7: Let V be a finite dimensional complex vector space with $\langle , \rangle$ a positive definite hermitian form on V. Let B be a basis for V such that $\langle , \rangle$ corresponds to the standard hermitian dot product (i.e. $\langle v, w\rangle = (B_v)^* B_w$). Let $P \in GL_n(\mathbb{C})$ be a change of basis matrix from B' to a basis B. Show that $\langle v, w\rangle = (B'_v)^* B'_w$ if and only if P is unitary.

Problem 8: Show that if M is a real normal $n \times n$ matrix and O is any $n \times n$ orthogonal matrix then M conjugated by O is a real normal $n \times n$ matrix.