

Math 151 Practice Midterm 1:

Rules: Write all answers in the spaces provided below. Do NOT separate the sheets. No notes or calculators allowed. All problems are worth the same amount. Make sure that in each problem you have at least one sentence explaining the key idea. Good Luck!

Name: _____

Problem 1: Is it the case that for all rational numbers x with $x < 1$, $\frac{1}{x} > 1$? If so prove it, if not characterize those rational x such that $\frac{1}{x} > 1$.

Problem 2: There are two fractions such that their difference is $\frac{4}{5}$ of the smaller one and their sum is 1 more than $\frac{3}{2}$ of the larger one. What are the two fractions?

Problem 3: Express the following as a fraction $\frac{a}{b}$ where a and b are whole numbers.

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$$

Problem 4: Suppose Allan, Bob and Christina each paint a house at a constant rate (but the rate for each may be different). When Alan paints the house it takes him 2 hours. When Bob paints the same house it takes him 3 hours. And when Christina paints the same house it takes her 4 hours. If all three of them paint the same house together (such that there is no overlap in what each paints) how long will it take them?

Problem 5: Convert each of the following to a finite decimal.

(a) $\frac{1.8}{0.03}$

(b) $6\frac{1}{5} \times 2\frac{1}{2}$

(c) $2\frac{1}{9} \times 3\frac{3}{5} + (3.2 - 2.002)$

(d) $\frac{1}{5} \times (0.22 + 0.022 + 0.0022)$

- Problem 6: (a) Prove $|x| = |y|$ if and only if $x^2 = y^2$.
- (b) For which whole numbers $n > 0$ does $|x| = |y| \Rightarrow x^n = y^n$?
Prove your answer (i.e. prove that for every whole number in your collection the implication holds and for every whole number not in your collection the implication does not hold).
- (b) Prove $\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$.

Problem 7: Find all rational numbers x such that $\frac{1}{2}|\frac{5}{3}x + 2| + 3 \geq 4$

Problem 8: Suppose x and y are rational numbers. Consider the following four numbers.

$$- (x + y)^2$$

$$- |x^2 + 2xy + y^2|$$

$$- x^2 + |2xy| + |y^2|$$

$$- 2(x^2 + y^2)$$

Can you rank these from least to greatest with just the information shown? If so rank them and explain your answer. If not give as many comparisons as you can and explain what information is missing. In addition give any equalities which must exist (if there are any).