

## **Instructions**

Dear Respondent:

Attached are a set of scenario questions that the Cal Teach research and evaluation team has developed to examine prospective teachers' understanding of three mathematical concepts for the purpose of teaching secondary students (i.e., grades 6-12).

Pages 3 to 12 list each question with space for you to write down your response. Feel free to write on the back of pages and/or add extra sheets as needed. Please provide **detailed explanations**, including the specific teaching methods you would use and how you would implement them in each scenario (e.g., including a table with specific values and explaining how the table would be used to introduce the concept of graphing a linear equation).

Please work alone and do not discuss these questions with other people. You may consult resources such as textbooks and websites as you respond to the questions. We ask that you indicate what resources were used and share any other comments you might have on the last page of this assignment.

Your responses will remain confidential and only be used for research and evaluation purposes. As you progress through Cal Teach, you will have opportunities to revisit and revise your responses. Your ongoing participation will allow us to study how Cal Teach students' mathematical understanding for teaching develops through the program.

If you have any clarifying questions while responding to the scenario questions, please feel free to contact Rebecca Poon at [rcpoon@berkeley.edu](mailto:rcpoon@berkeley.edu).

Since these questions are still in development, we ask that you do **NOT** share, disseminate, or use them without our knowledge.<sup>1</sup>

Thank you very much,

Xiaoxia Newton & Nicci Nunes  
Cal Teach Berkeley

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<sup>1</sup> We thank those who have read earlier drafts of the instrument and shared with us constructive feedback, in particular, Hung-Hsi Wu, Alan Schoenfeld, and George Gagnon. However, Xiaoxia Newton and Rebecca Poon are responsible for any errors.

**First, we would like to gather some basic information about you.**

Your name: \_\_\_\_\_

Current major(s) (mark all that apply):

- Biology
- Chemistry
- Computer Science
- Earth Science
- Engineering
- Environmental Science
- Mathematics
- Mathematics with a Teaching Concentration
- Mathematics, Applied
- Physics
- Statistics
- Undecided
- Other (specify): \_\_\_\_\_

Are you a transfer student?

- Yes, from (specify prior college): \_\_\_\_\_
- No

Number of years you have attended UC Berkeley (counting the current school year as one):

- One
- Two
- Three
- Four
- Five
- Six

Would you be willing to participate in a follow-up interview?

- Yes
- No

### Scenario 1A: Dividing Fractions

Imagine that you are teaching a group of *sixth graders* to solve the following problem:<sup>2</sup>

A rope  $43\frac{3}{8}$  meters long is cut into pieces which are each  $\frac{5}{3}$  meters long. How many such pieces can we get out of the rope?

1. Describe how you would approach this problem with your students in a way that builds on their prior mathematical knowledge and/or skills related to division of whole numbers.
2. Suppose your answer is  $A\frac{B}{C}$  where  $A$ ,  $B$ , and  $C$  are whole numbers so that  $0 < B < C$  and  $C \neq 0$ . How would you explain to students what the  $A$  means and what the  $\frac{B}{C}$  means?

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<sup>2</sup> The math problem is adapted from *Lecture Notes for the 2009 Pre-Algebra Institute* (Wu, 2009).

### Scenario 1B: Dividing Fractions

Imagine that one of your *sixth graders* missed your lesson on fraction division. He studies the textbook and attempts the homework on his own. The next day he comes and tells you, “The book says  $\frac{17}{8} \div \frac{5}{9} = \frac{17}{8} \times \frac{9}{5}$ , but I don’t understand why.” How would you respond to this student?

## Scenario 2A: Linear Equation in Two Variables and its Graph

You are giving your *seventh graders* the district pre-algebra common assessment. One word problem on the assessment reads as follows:

“Abe is training for the Boston marathon, so he runs every day. This morning he ran 4 miles in 40 minutes. How many miles did he run in the first 20 minutes? Show your work.”

One of your top-performing students in the class, nicknamed “Einstein,” comes to you with her test and says, “There is not enough information given to solve this problem.”

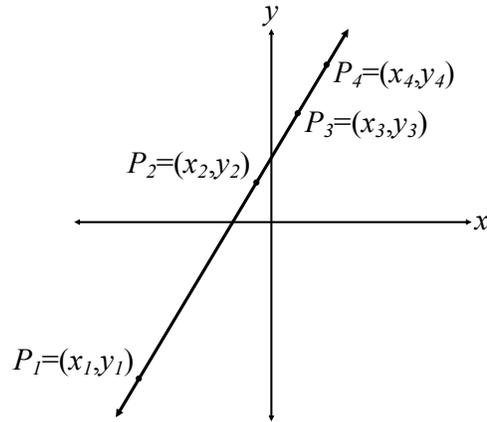
1. How would you respond to “Einstein?”
2. Explain how you would teach the class about the critical mathematical concept(s) for solving this kind of word problem.

## Scenario 2B: Linear Equation in Two Variables and its Graph

You are teaching a class of *eighth graders* and writing a test to assess their ability to translate word problems into symbolic equations. Come up with a story that could be represented by the equation  $y = \frac{7}{5}x + 4$ .

### Scenario 2C: Linear Equation in Two Variables and its Graph

How would you help eighth graders understand that the slope of a non-vertical line can be calculated using any two distinct points on the line (e.g., the slope of the line below can be calculated with points  $P_1$  and  $P_2$  or points  $P_3$  and  $P_4$ )?



## Scenario 2D: Linear Equation in Two Variables and its Graph

Suppose you are teaching a class of *ninth graders* about graphs of linear equations in two variables. Explain how you would help students see the connection among:

- the standard form  $ax + by = c$  where  $a, b, c$  are constants and  $a \neq 0$  or  $b \neq 0$
- the slope-intercept form  $y = mx + k$  where  $m, k$  are constants
- the point-slope form  $y - q = m(x - p)$  where  $p, q$  are constants
- the graph of a linear equation

### Scenario 3: Quadratic Function, its Graph, and the Quadratic Formula

You and your colleagues are discussing struggles your *10th graders* have in learning quadratic functions. As a group, you are trying to figure out how best to help students overcome the struggles. You decide that one approach is to help students “see” the connection between the quadratic function and its graph.

1. Let  $g$  and  $f$  be the quadratic functions  $g(x) = x^2$  and  $f(x) = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are constants and  $a \neq 0$ . Describe how you would use the graph of  $g$  to help students see how one arrives at the graph of  $f$ .

**Scenario 3: Quadratic Function, its Graph, and the Quadratic Formula (continued)**

2. How would you help students understand that  $f(x) = ax^2 + bx + c$  can be re-written into the forms  $a(x - h)^2 + k$  and  $a(x - p)(x - q)$  where  $h, k, p,$  and  $q$  are suitable constants and assuming the discriminant  $b^2 - 4ac \geq 0$ ? How would you explain the relationship among the constants  $a, b, c, h, k, p,$  and  $q$ ?

**Scenario 3: Quadratic Function, its Graph, and the Quadratic Formula (continued)**

3. After you introduce the three different forms of the quadratic function to your class, one student asks, “What’s the point of writing the same quadratic function into different forms?” Describe how you would respond to this student’s question.

**Scenario 3: Quadratic Function, its Graph, and the Quadratic Formula (continued)**

4. How would you help students understand that each form of the quadratic function is useful for visualizing the graph of the quadratic function?

**Finally, we'd like to hear any comments you might have about the scenario questions. In addition, please indicate any resources you may have used to respond to the questions. For published materials, include *title, publisher, and year* (e.g., Algebra 1 McDougal Littel 2007); for websites, include *url address* (e.g., <http://www.math.ubc.ca/~morey/java/pyth/>).**

Scenario	Question	Comments and Resource(s)
1A	1	Comments:
		Resource(s):
	2	Comments:
		Resource(s):
1B		Comments:
		Resource(s):
2A	1	Comments:
		Resource(s):
	2	Comments:
		Resource(s):
2B		Comments:
		Resource(s):

Scenario	Question	Comments and Resource(s)
2C		Comments:
		Resource(s):
2D		Comments:
		Resource(s):
3	1	Comments:
		Resource(s):
	2	Comments:
		Resource(s):
	3	Comments:
		Resource(s):
	4	Comments:
		Resource(s):