Homework 3

Solutions

1)

$\frac{2}{2}$ 1. §1.6 #3

Let A, B be fractions and $B \neq 0$. Prove that for any nonzero whole number j $\underbrace{\frac{A}{B} + \frac{A}{B} + \cdots + \frac{A}{B}}_{j} = \frac{jA}{B}$

Let $A = \frac{k}{\ell}$ and $B = \frac{m}{n}$

So, $\underbrace{\frac{\frac{k}{\ell}}{\frac{m}{n}} + \frac{\frac{k}{\ell}}{\frac{m}{n}} + \cdots}_{j} + \frac{\frac{k}{\ell}}{\frac{m}{n}} = \underbrace{\frac{kn}{\ell m} + \frac{kn}{\ell m} + \cdots \frac{kn}{\ell m}}$ Definition of Division on Fractions

$= \frac{kn + kn + \cdots + kn}{\ell m}$ Definition of addition of Fractions

good.

$= \frac{jkn}{\ell m} = j \cdot \frac{k}{\ell} \cdot \frac{n}{m}$ Definition of multiplication of Whole #s

$= j \cdot A \cdot \frac{1}{B} = \frac{jA}{B}$ ✓ Substitution

2. §1.6 #5

2)

1.6, #5 (2) We want $\frac{2}{7}$ in two parts A, B so that "$\frac{A}{B} = \frac{4}{5}$".

$\frac{2}{2}$

Well,

$$\frac{A}{B} = \frac{4}{5} \Rightarrow 5A = 4B \quad \text{(laws of fractions)}$$

$$\Rightarrow \frac{1}{5} \cdot 5A = \frac{1}{5} \cdot 4B \quad \text{(multiply both sides by } \tfrac{1}{5}\text{)}$$

great.

$$\Rightarrow \frac{5A}{5} = \frac{4B}{5} \quad \text{(multiplication).}$$

$$\Rightarrow A = \frac{4B}{5} \quad \text{(cancellation law).}$$

We know that, $\frac{2}{7} = A + B$ (given)

$$\Rightarrow \frac{2}{7} = \frac{4B}{5} + B \quad \left(A = \frac{4B}{5} \right)$$

[OVER]

$$\Rightarrow \frac{2}{7} = \frac{4B}{5} + B \quad (A = \frac{4B}{5})$$

$$\Rightarrow \frac{2}{7} = \frac{4B}{5} + \frac{5B}{5} \quad (\text{multiply by } \frac{5}{5} = 1)$$

$$\Rightarrow \frac{2}{7} = \frac{9B}{5} \quad (\text{addition of fractions})$$

$$\Rightarrow 10 = 63B \quad (\text{law of fractions})$$

$$\Rightarrow 10 \cdot \frac{1}{63} = \frac{1}{63} \cdot 63B \quad (\text{multiply both sides by } \frac{1}{63}).$$

$$\Rightarrow \frac{10}{63} = \frac{63B}{63} \quad (\text{multiplication})$$

$$\Rightarrow \frac{10}{63} = B \quad (\text{cancellation law}).$$

Now... $A = \frac{4B}{5}$

$$= \frac{4}{\cancel{5}_1} \cdot \frac{\cancel{10}^2}{63} \quad (\text{substitution})$$

$$= \frac{8}{63} \quad (\text{multiplication})$$

Therefore, $\boxed{A = \frac{8}{63} \ \& \ B = \frac{10}{63}}$

3)

2) Helena drives from A to B at X mph and back at Y mph.

(a) What is her average speed for the round trip?

Let the distance from A to B be δ. Also, let the travel time from A to B be t_1 and the travel time from B to A be t_2. Then we know that $\delta = Xt_1$ and $\delta = yt_2$. Thus, solving for times we get $t_1 = \delta/x$ and $t_2 = \delta/y$. We know that average speed is $\frac{\text{distance traveled}}{\text{travel time}}$. Thus, using this formula, we have average speed $= \frac{2d}{t_1+t_2} = \frac{2\delta}{\frac{\delta}{x}+\frac{\delta}{y}}$.

Notice that the distance traveled is 2δ because we are traveling to AND from A and B. Also notice that we are adding the times in the denominator because we want the average speed over the entire trip which is traveled in time t_1+t_2. Now, we can add fractions and use the inverse multiplication rule for division to get:

$$\frac{2\delta}{\frac{\delta}{x}+\frac{\delta}{y}} = \frac{2\delta}{\frac{\delta y+\delta x}{xy}} = \frac{2\delta}{\frac{\delta(y+x)}{xy}} = \frac{2\delta}{1}\cdot\frac{xy}{\delta(y+x)} = \frac{2xy\cdot\delta}{(x+y)\cdot\delta} = \frac{2xy}{(x+y)}$$

Where the last equality holds because of the cancellation law. Thus, the average speed is $2xy/(x+y)$. great

(b) If it takes t total hours to travel to and from A and B, how far apart are the towns?

Using the same definitions from part a, let $t = t_1+t_2 = \frac{\delta}{x}+\frac{\delta}{y} = \delta(\frac{1}{x}+\frac{1}{y})$. Thus, solving for δ, we get that

$$\delta = \frac{t}{(\frac{1}{x}+\frac{1}{y})} = \frac{t}{(\frac{y+x}{xy})} = \frac{t}{1}\cdot\frac{xy}{(y+x)} = \frac{txy}{(x+y)}$$ where

the second equality holds by our algorithm for adding fractions and the third equality holds by the inverse multiplication rule for division.

perfect.

4)

Ex. 4 (1.7, #3) $\left(\frac{2}{2}\right)$

(1) From the first given statement, we can deduce the following eq

$v_j - (45\% \text{ of } v_j) = N$, where V_J is the value in june, and N is the present value.

$\Leftrightarrow v_j - \left(\frac{45}{100} \cdot v_j\right) = N$ by def. of a percent

$\Leftrightarrow \frac{v_j(100)}{100} - \frac{45v_j}{100} = N$ by cancellation law (Assume $N \neq 0$)

$\Leftrightarrow \frac{100v_j - 45v_j}{100} = N \Leftrightarrow \frac{55v_j}{100} = N \Leftrightarrow \frac{11v_j}{20} = N$ (I)

[since $\frac{(11 \cdot 5)v_j}{(20 \cdot 5)}$ by cancellation law]

From the second given statement,

$N + (60\% \text{ of } N) = V_j \Leftrightarrow N + \left(\frac{60}{100} \cdot N\right) = v_j$

$\Leftrightarrow \frac{100N + 60N}{100} = v_j \Leftrightarrow \frac{160N}{100} = v_j \Leftrightarrow \frac{8}{5}N = v_j$ (II)

[Since $\frac{160}{100} = \frac{8 \cdot 20}{5 \cdot 20}$ by cancellation law]

(2) Next, we check if the two statements are true by substituting (II) into (I).

$\frac{11v_j}{20} = N \Leftrightarrow \frac{11}{20}\left(\frac{8}{5}N\right) = N$ by substitution

perfect.

$\Leftrightarrow \frac{11 \cdot 8 \cdot \cancel{N}}{5 \cdot 20} = \cancel{N}$ by invert & multiply rule
by cancellation law

$\Leftrightarrow \frac{88}{100} \neq 1$

Hence, the stock broker is $\boxed{\text{incorrect}}$ since his statements are mathematically unequivalent. ☑

3) Moreover, let $P =$ % of the stock's rise at value \$N in order to regain former val

So $N + (P\% \text{ of } N) = v_j \Leftrightarrow N + \left(\frac{P}{100} \cdot N\right) = v_j \Leftrightarrow \frac{100N + PN}{100} = v_j$ by def. of a percent

Recall (I) $\frac{11v_j}{20} = N \Leftrightarrow$ so $\frac{11\left(\frac{100N + PN}{100}\right)}{20} = N$ by substitution $\Leftrightarrow \frac{11(\cancel{N})(100+P)}{100 \cdot 20} = \cancel{N} = 1$ by factoring/canceling

$\Leftrightarrow 11(100+P) = 100 \cdot 20$ (by cross multiplication)

$\Leftrightarrow (100+P) = \frac{100 \cdot 20}{11} = \frac{2000}{11}$ by subtraction

$\Leftrightarrow P = \frac{2000}{11} - 100 = \frac{2000 - 1100}{11} = \frac{900}{11}$ hence $\Longrightarrow$

the stock must rise $\left(\frac{900}{11}\right)\%$ of its present value to regain former value

5)

1 a) To find the midpoint C, we solve $B-C=C-A$ for C. So $B=2C-A$ and $A+B=2C$ so $\boxed{\frac{A+B}{2}=C}$ which is the midpoint.

b) Find point D so that ratio of length $[A,D]$ to $[D,B]$ is 2 to 5 so $D-A$ is the length (A,D) and $B-D$ is the length $B-D$ so $\frac{D-A}{B-D}=\frac{2}{5}$ $= 2B-2D=5D-5A$

$7D=2B+5A$

and $\boxed{D=\frac{2B+5A}{7}}$

$\frac{2}{2}$

c) So we have two lengths $E-A$ and $B-E$ and their ratio is $m:n$ so we have

$\frac{E-A}{B-E}=\frac{m}{n}$ cross multiply and manipulate to solve for E.

$En-An=Bm-Em$

$E(n+m)=Bm+An$

$\boxed{E=\frac{Bm+An}{n+m}}$ and this is the general case.

good.

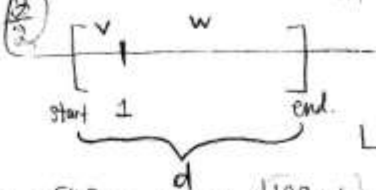
6)

6. Constant speed = v $t_{old} = 5$ hours. $v = \frac{d}{t}$, $d = vt$

2/2

at $t = 1$ constant speed = w $t_{new} = 4\frac{1}{2}$ hours

↳ so went w for $3\frac{1}{2}$ hours.



$$d = 5v$$

$$d = 1(v) + 3\tfrac{1}{2}(w) = v + \tfrac{7}{2}w$$

↳ $5v = v + \frac{7}{2}w \Rightarrow w = \frac{(4v)2}{7} = \frac{8v}{7} = \frac{8(\frac{100}{7})v}{7(\frac{100}{7})} = \frac{\frac{800}{7}}{100}v$

so $\frac{800}{7}\% - 100\% = \boxed{\frac{100}{7}\%}$ $\Rightarrow$ w is $\frac{800}{7}\%$ of v   perfect (v)

7)

11) a) For someone to paint a house at a constant rate mean that for every fixed interval of time put in, the exact same area of the house is painted in each fixed interval with no exception. good.

b) So max takes 90 hours to paint the house alone, so the Area of the house can equal A, the time it takes is 90 so Max's rate m is $\frac{A}{90} = m$

2/2

Nancy takes an unknown amount of time t, which we need to solve for t. So Nancy's rate $\frac{A}{t} = n$

We know that $m + n = A$ since both their rates added together and divided by the area would equal the time, 56 hours, so

$$\frac{56A}{90} + \frac{56A}{t} = A$$

(we multiply both by 56 because it takes both 56 hours to complete, so if Max paints m square feet in 56 hours, the by definition $\frac{m}{56} = \frac{A}{90t}$ and similarly $\frac{n}{56} = \frac{A}{t}$)

So $A\left(\frac{56}{90} + \frac{56}{t}\right) = A$

Cancellation law

and $\frac{56}{90} + \frac{56}{t} = 1$

use addition of fractions

$\frac{56t + 56 \cdot 90}{90t} = 1$ cross multiply

$56t + 5040 = 90t$ Solve for t

$5040 = 34t$

$t = \frac{5040}{34} = 148\frac{4}{17}$ hours to paint herself.

Good.

8)

a) The initial deposit is \$93.00. We must go year by year. So for year one we need to add 6% of \$93.00 to \$93.00. This is the same as saying we want 106% of \$93.00.

$\frac{2}{2}$

So $\frac{106}{100}$ (106% by definition) times 93 should equal our new balance after year 1. So $\frac{106}{100} \cdot 93 = \frac{9858}{100} = \$98.58$

Now repeat this algorithm for year 2, replacing \$93.00 with \$98.58 $\frac{106}{100} \cdot \$98.58 = \frac{10449.48}{100} = \$104.49$

And again replacing \$98.58 with \$104.49

$\frac{106}{100} \cdot \$104.49 = \frac{11075.94}{100} = \$110.76$. This is the amount in the account after 3 years.

b) So for n years lets think about what we did. We first took our initial deposit then multiplied by $\frac{106}{100}$, got that answer then multiplied again by $\frac{106}{100}$ and again and so forth. So for n years we would take our initial deposit, \$93.00 and multiply it by $\frac{106}{100}$, n times. So the amount \$M is

$$\$M = \$93.00 \times \underbrace{\left(\frac{106}{100} \times \frac{106}{100} \cdots \frac{106}{100}\right)}_{n \text{ times}}$$

good

9)

2.1

2/2

1) Show that between any two rational numbers there is another rational number.

Let A, B be rational numbers, i.e they have a mirror reflection of and A^* and B^* on the opposite side of zero and are equidistant from 0.

Consider three cases:

(case 1) $A < 0 < B$, 0 is a rational number so we are done

(case 2) $0 \leq A < B$, we shown that a rational number exists in Hw 1.2 #13 so this is also true.

(case 3) $A < B \leq 0$

then we know that by de of rational numbers $\exists$ A^*, B^* such that

$0 \leq B^* < A^*$. Now let C be a point on the number line such that

$0 \leq B^* < C < A^*$. This holds since we can find a point between B^* and A^* that is a fraction. Since C is a rational number, C^* is a reflection equidistant on the other side of 0.

So $0 \leq B^* < C < A^*$

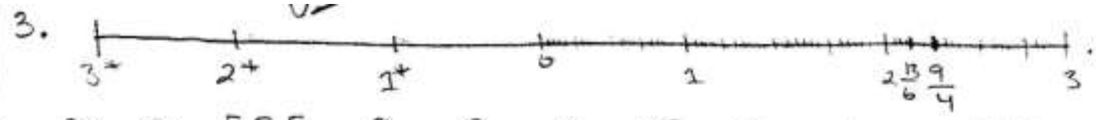
$A < B \leq 0 \leq B^* < C < A^*$ perfect.

$A < C^* < B \leq 0$

so C^* is a rational number between A < B.

10)

3.



good

By the FFF $\frac{9}{4}$, $\frac{13}{6}$, $\frac{11}{5}$, $\frac{15}{7}$ can be written with a common denominator. Let it be 420.

$\frac{9}{4} = \frac{945}{420}$ $\frac{13}{6} = \frac{910}{420}$ $\frac{11}{5} = \frac{924}{420}$ $\frac{15}{7} = \frac{900}{420}$

Because $\left(\frac{15}{7}\right)^*$ and $\frac{15}{7}$ are mirror reflections they are the same distance from 0. Same for $(11/5)^*$ and $11/5$.

Because $\frac{900}{420}$ is the smallest fraction, $\frac{15}{7}$ is closest to 0.