Homework 2

Solutions

1)

Math 151 HW 2

$\frac{2}{2}$

1a $\left(12\frac{2}{3}\times 12\frac{2}{3}\times 12\frac{2}{3}\right)\times\left(2\frac{1}{19}\times 2\frac{1}{19}\times 2\frac{1}{19}\right)\times\left(\frac{1}{26}\right)$

$=\left(\frac{38}{3}\times\frac{38}{3}\times\frac{38}{3}\right)\times\left(\frac{39}{19}\times\frac{39}{19}\times\frac{39}{19}\right)\times\frac{1}{26}$

$=\frac{\cancel{19}\times 2\times\cancel{19}\times 2\times\cancel{19}\times\cancel{2}}{\cancel{3}\times\cancel{3}\times\cancel{3}}\times\frac{\cancel{3}\times 13\times\cancel{3}\times 13\times\cancel{3}\times\cancel{13}}{\cancel{19}\times\cancel{19}\times\cancel{19}}\times\frac{1}{\cancel{2}\times\cancel{13}}$

$= 2\times 2\times 13\times 13 = 676$

b. $\left(\frac{7}{18}\times 4\frac{2}{3}\right)+\left(2\frac{1}{6}\times\frac{7}{18}\right)+\left(\frac{7}{18}\times 3\frac{1}{6}\right)$

$=\left(\frac{7}{18}\times\frac{14}{3}\right)+\left(\frac{13}{6}\times\frac{7}{18}\right)+\left(\frac{7}{18}\times\frac{19}{6}\right)$

$=\frac{98}{54}+\frac{91}{108}+\frac{133}{108}=\frac{98\times 2}{54\times 2}+\frac{91}{108}+\frac{133}{108}$

$=\frac{196+91+133}{108}=\frac{420}{108}=\frac{4\cdot 105}{4\cdot 27}=\frac{105}{27}$

c $8\frac{2}{50}\times 1250\frac{1}{2}=\frac{402}{50}\times\frac{2501}{2}=\frac{201\times\cancel{2}\times 2501}{50\times\cancel{2}}$

$=\frac{502,701}{50}$

solid.

2)

Ex.2) (1.4, #3) (c) distributive: $\left(\frac{k}{\ell}+\frac{m}{n}\right)\times\frac{a}{b}=\left(\frac{k}{\ell}\times\frac{a}{b}\right)+\left(\frac{m}{n}\times\frac{a}{b}\right)$

Given 3 fractions: $\frac{k}{\ell}$, $\frac{m}{n}$, and $\frac{a}{b}$; $\ell, n, b \neq 0$

By fraction addition | $\left(\frac{k}{\ell}+\frac{m}{n}\right)\times\frac{a}{b}=\left(\frac{kn+m\ell}{\ell n}\right)\times\frac{a}{b}$

By product rule | $=\frac{(kn+m\ell)a}{(\ell n)b}$

By the distributive law of whole #'s | $=\frac{kna+m\ell a}{\ell nb}=\frac{k\cancel{n}a}{\ell\cancel{n}b}+\frac{m\cancel{\ell}a}{\cancel{\ell}nb}\overset{\text{by cancellation law}}{=}\frac{ka}{\ell b}+\frac{ma}{nb}$

By product rule | $=\left(\frac{k}{\ell}\times\frac{a}{b}\right)+\left(\frac{m}{n}\times\frac{a}{b}\right)$ ☑

Ex.2) (1.4, #3) 2/2) Give a detailed proof of the corollary: the mult. of fractions is associative, commutative, and distributive.

(a) commutative: $\frac{k}{\ell}\times\frac{m}{n}=\frac{m}{n}\times\frac{k}{\ell}$

Given 2 fractions: $\frac{k}{\ell}$ and $\frac{m}{n}$, ℓ and $n \neq 0$; $k,\ell,m,n \in \mathbb{N}$

By thm. 1 (the product formula): $\frac{k}{\ell}\times\frac{m}{n}=\frac{km}{\ell n}$ good.

Since the (x) of whole #'s is commutative, Always make your assumptions clear - I like it.

$\frac{k\times m}{\ell\times n}=\frac{m\times k}{n\times\ell}=\frac{m}{n}\times\frac{k}{\ell}$ ☑

(b) associative: $\left(\frac{k}{\ell}\times\frac{m}{n}\right)\times\frac{a}{b}=\frac{k}{\ell}\times\left(\frac{m}{n}\times\frac{a}{b}\right)$

Given 3 fractions: $\frac{k}{\ell}$, $\frac{m}{n}$, and $\frac{a}{b}$, ℓ and n and $b \neq 0$

By the product formula: $\left(\frac{k}{\ell}\times\frac{m}{n}\right)\times\frac{a}{b}=\frac{(km)a}{(\ell n)b}$

Since the (x) of whole #'s is associative,

$\frac{(km)a}{(\ell n)b}=\frac{k(ma)}{\ell(nb)}=\frac{k}{\ell}\times\frac{(ma)}{(nb)}=\frac{k}{\ell}\times\left(\frac{m}{n}\times\frac{a}{b}\right)$ ☑

(c) distributive: (next page)

3)

* Ex. 3) (1.4 #5) $\frac{2}{3}$

Show that given a fraction A and a fraction L,

(a) $\exists$ a rectangle with area equal to A but with a perimeter that is bigger than L.

(1) Draw a diagram & state given conditions:

$hw = \text{Area} = A$

$2h+2w = \text{Perimeter} > L$

(2) Fix h to find w in terms of h s.t. Area = A $(w, h > 0)$

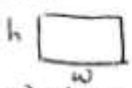
$\Rightarrow h \cdot w = A \Rightarrow h \cdot \frac{A}{h} = A \Rightarrow w = \frac{A}{h}$

(3) $2h+2w \Rightarrow 2h + 2\left(\frac{A}{h}\right) > L$ iff $h > L$ ☑ .

$\boxed{\text{Hence, a rectangle with height } h > L \text{ and width } w = \frac{A}{h} \text{ satisfies the given condition.}}$

(b) $\exists$ a rectangle with perimeter equal to L but Area < A.

(1) Draw a diagram & state given conditions:



$hw = \text{Area} < A$

$2h+2w = \text{Perimeter} = L$

(2) Solve for h and find the Area in terms of L and w.

$2h+2w = L \Rightarrow h = \frac{L-2w}{2}$. Thus $\text{Area} = hw = \left(\frac{L-2w}{2}\right) w = \frac{Lw}{2} - w^2 < A$

(3) Hence, $\left(\frac{Lw}{2} - w^2\right) < A \Leftrightarrow \frac{Lw}{2} < A \Leftrightarrow w < \frac{2A}{L}$

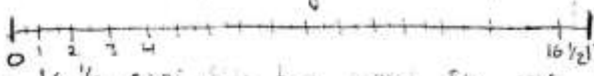
So $\text{Area} < \frac{L\left(\frac{2A}{L}\right)}{2} - \left(\frac{2A}{L}\right)^2 \Leftrightarrow \text{Area} < \left(A - \frac{4A^2}{L^2}\right) < A$ ☑

$\boxed{\text{Hence, a rectangle with width } w < \frac{2A}{L} \text{ and height } h = \frac{L-2w}{2} \text{ satisfies condition.}}$

4)

$\frac{2}{3}$

6. $16\frac{1}{2}$ cups fill the punch bowl and 1 cup is $9\frac{1}{3}$ ounces. Find the capacity of the punch bowl?

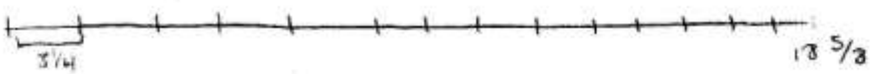
Solution: let  Represent the punch bowl. $16\frac{1}{2}$ cups fill the bowl so we will divide the line into $16\frac{1}{2}$ parts. And each cup is $9\frac{1}{3}$ oz. So each part represents $9\frac{1}{3}$ oz. So we have $\underbrace{9\frac{1}{3} + 9\frac{1}{3} + 9\frac{1}{3} + 9\frac{1}{3} \ldots}_{16\frac{1}{2} \text{ times.}}$

$$\therefore \quad 9\frac{1}{3} \times 16\frac{1}{2} = \frac{28}{3} \times \frac{33}{2} = \frac{(28)(33)}{(3)(2)} =$$

perfect.

$$\frac{(28)(33)}{(2)(3)} = \frac{28}{2} \times \frac{33}{3} = \frac{14}{1} \times \frac{11}{1} = 154 \text{ oz.}$$

(b) A rod can we cut into $18\frac{5}{8}$ of a short piece that is $3\frac{1}{4}$



So we have $18\frac{5}{8}$ sections that are $3\frac{1}{4}$ long.

so to get the length of the rod we would add $\underbrace{3\frac{1}{4} + 3\frac{1}{4} \ldots}_{18\frac{5}{8} \text{ times}}$ which is the definition of multiplication. So we can write $3\frac{1}{4} \times 18\frac{5}{8}$, $3\frac{1}{4} = \frac{13}{4}$, $18\frac{5}{8} = \frac{149}{8}$

$$\frac{13}{4} \times \frac{149}{8} = \frac{(13)(144)}{(4)(8)} = \frac{1437}{32}$$

good.

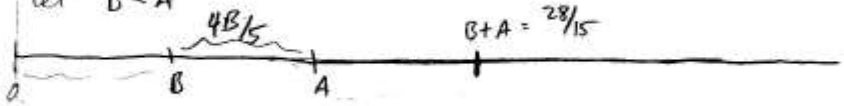
5)

1.4.11

$\frac{2}{2}$

Given A, B are fractions. $A + B = \frac{28}{15}$, $A - B = B \times 4/5$

let $B < A$



$B + A - 2B = A - B \rightarrow \frac{28}{15} - 2B = \frac{4B}{5}$ place all terms in common denom

$\frac{28}{15} - \frac{30B}{15} = \frac{12B}{15} \Rightarrow 28 - 30B = 12B \rightarrow B = 28 \div 42 = \boxed{28/42}$ or $2/3$

$A + B = 28/15 \rightarrow A + \frac{28}{42} = \frac{28}{15}$ place all terms in common denom

$\frac{210A}{210} + \frac{140}{210} = \frac{392}{210} \rightarrow 210A + 140 = 392 \rightarrow A = \boxed{\frac{252}{210}}$ or $6/5$

good.

6)

a) 2 tablespoons to dechlorinate 120 gal of water. 3 teaspoons = 1 tablespoon

2/2 how many tea spoons of chemical are needed to dechlorinate x gallons of water.

1 tablespoons = 6 teaspoons to dechlorinate 120 gallons of water

so $\frac{120}{6} = D = 20$.

so if there is x amount of water, then you would need

$\frac{x}{c} = 20 \Rightarrow c = \frac{x}{20}$ teaspoons of chemral.

good.

7)

7. Let $a = qd + r$, $a = Qd$

$\frac{2}{3}$ $\Rightarrow Qd = qd + r$ Q is the mixed fraction w/ whole number

$Q = \frac{qd+r}{d} = q + \frac{r}{d}$ q and numerator r. good

quotient remainder

8)

8. (a) $\frac{5}{12}$ of a sack of rice is $8\frac{2}{3}$ the weight of 5 books. Each book weighs $2\frac{1}{2}$ lbs. How much does 1 sack of rice weigh?

$\Rightarrow$ 5 books that weigh $2\frac{1}{2}$ lbs each is $\underbrace{2\frac{1}{2}+2\frac{1}{2}\cdots}_{5\text{ times}}$
which is multiplication so, $5 \times 2\frac{1}{2} = \frac{25}{2}$

Now $8\frac{2}{3}$ of 5 books is also multiplication, so

$$8\frac{2}{3} \times \frac{25}{2} = \frac{26}{3} \times \frac{25}{2} = \frac{(26)(25)}{(3)(2)} = \frac{325}{3}$$

$\therefore \frac{5}{12}$ of a sack of rice weighs $\frac{325}{3}$ lbs.

let m/n = 1 sack of rice so

$$\frac{5}{12} \text{ of } \frac{m}{n} = \frac{325}{3} \Rightarrow \frac{5}{12} \times \frac{m}{n} = \frac{325}{3}$$

$$\frac{m}{n} = \frac{325}{3} \times \frac{12}{5} \Rightarrow \frac{m}{n} = \frac{(325)(12)}{(3)(5)} = 260 \quad \therefore \frac{m}{n} = 260$$

So 1 sack of rice weighs 260 lbs.

(b) charges $\frac{m}{n} \times 8$ dollars for $\frac{m}{n}$ of a small pizza. Special sale gets $\frac{1}{2}$ of a pizza for the price of $\frac{1}{3}$. How much would $8\frac{2}{3}$ small pizzas cost.

$\Rightarrow$ Normally $\frac{m}{n}$ of pizza cost 8 dollars. But with the sale, $\frac{1}{2}$ pizza costs $\frac{1}{3} \times 8$. So,

$8\frac{2}{3} \div \frac{1}{2} \times \frac{8}{3} =$ this can divide $8\frac{2}{3}$ by $\frac{1}{2}$ to get the number of halves of pizza are in $8\frac{2}{3}$ then multiplies that number by $\frac{8}{3}$ which is the cost. Therefore,

$$\left(8\frac{2}{3} \div \frac{1}{2}\right) \times \frac{8}{3}\text{dollars} = \frac{26}{3} \times 2 \times \frac{8}{3} = \frac{416}{9}$$

$\therefore$ the price of $8\frac{2}{3}$ pizza is $46\frac{2}{9}$ dollars.

good

9)

2

a) $\frac{1}{\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)} = ?$ $\quad \frac{1}{\frac{1}{2}\left(\frac{1}{2/3}+\frac{1}{5/4}\right)} = ?$

$$\frac{1}{\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)} = \frac{1}{\frac{1}{2}\left(\frac{4}{12}+\frac{3}{12}\right)} = \frac{1}{\frac{1}{2}\left(\frac{7}{12}\right)} = \frac{1}{\frac{7}{24}} \boxed{= \frac{24}{7}}$$

adding fractions — multiplying fractions — dividing

$$\frac{1}{\frac{1}{2}\left(\frac{1}{2/3}+\frac{1}{5/4}\right)} = \frac{1}{\frac{1}{2}\left(\frac{5/4}{2/3\cdot 5/4}+\frac{2/3}{5/4\cdot 2/3}\right)} = \frac{1}{\frac{1}{2}\left(\frac{5/4+2/3}{5/6}\right)} = \frac{1}{\left(\frac{5/4+2/3}{5/3}\right)} =$$

9. $= \dfrac{\frac{5}{3}}{\frac{5}{4}+\frac{2}{3}} = \dfrac{\frac{5}{3}}{\frac{15}{12}+\frac{8}{12}} = \dfrac{\frac{5}{3}}{\frac{23}{12}} = \dfrac{5}{\cancel{3}_1} \times \dfrac{\cancel{12}^4}{23}$

$\boxed{= \dfrac{20}{23}}$

b) If x, y are nonzero fractions, what is $\dfrac{1}{\frac{1}{2}(\frac{1}{x}+\frac{1}{y})} = ?$
(This expression for x and y turns up often enough to merit a name the harmonic mean of x and y)

Let $\dfrac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)} = ?$

$= \dfrac{1}{\frac{1}{2}\left(\frac{y+x}{xy}\right)}$

$= 2\left(\dfrac{xy}{y+x}\right)$ (Definition of dividing fractions (invert & multiply))

$\boxed{= \dfrac{2xy}{y+x}}$

c) If x, y, u, v are nonzero fractions so that $x < u$ and $y < v$ prove that

$\dfrac{xy}{x+y} < \dfrac{uv}{u+v}$

So, multiplying both sides of the inequality would yield

$2\left(\dfrac{xy}{x+y}\right) < 2\left(\dfrac{uv}{u+v}\right)$

In part b, we found the previous form of this as:

$\dfrac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)} < \dfrac{1}{\frac{1}{2}\left(\frac{1}{u}+\frac{1}{v}\right)}$

So, using the cross-multiplication algorithm corollary:

$\frac{1}{2}\left(\frac{1}{u}+\frac{1}{v}\right) < \frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)$

good

Multiplying both sides by 2

$\dfrac{1}{u}+\dfrac{1}{v} < \dfrac{1}{x}+\dfrac{1}{y}$

Since $x < u \Rightarrow \frac{1}{u} < \frac{1}{x}$, and $y < v \Rightarrow \frac{1}{v} < \frac{1}{y}$

So, $\frac{1}{u} < \frac{1}{x}$

$\Leftrightarrow \frac{1}{u} + \frac{1}{v} < \frac{1}{x} + \frac{1}{v}$

And $\frac{1}{v} < \frac{1}{y}$

$\Leftrightarrow \frac{1}{v} + \frac{1}{x} < \frac{1}{y} + \frac{1}{x}$

So, $\frac{1}{u} + \frac{1}{v} < \frac{1}{x} + \frac{1}{v} < \frac{1}{y} + \frac{1}{x}$

And, $\frac{1}{u} + \frac{1}{v} < \frac{1}{y} + \frac{1}{x}$

Thus, we have proved that this statement is true.

10)

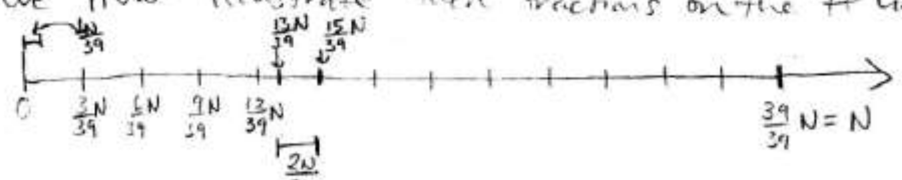
Ex. 10) (1.5, #10) (a) Use the # line to solve:

$\frac{2}{2}$ If $\frac{5}{13}$ of a number N exceeds a third of N by 8, what is N?

(1) In other words, the difference between "$\frac{5}{13}$ of N" and "$\frac{1}{3}$ of N" is 8. For clarity, we will force the denominators to be the same in order to better compare the fractions $\frac{1}{3}$ and $\frac{5}{13}$ on the # line.

Then, $\frac{1}{3} = \frac{1 \cdot 13}{3 \cdot 13} = \frac{13}{39}$ and $\frac{5}{13} = \frac{5 \cdot 3}{13 \cdot 3} = \frac{15}{39}$ by the equivalent fractions thm.

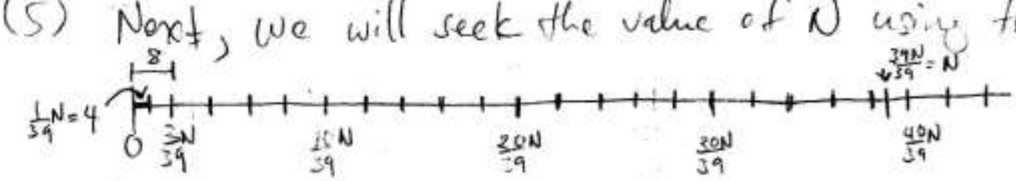
(2) We now illustrate these fractions on the # line:



(3) From the # line, we clearly see that the difference between $\frac{15N}{39}$ and $\frac{13N}{39}$ is the length between the two points, namely $\frac{2N}{39}$.

(4) From the given statement, we know that the difference of $\left(\frac{15N}{39} \text{ and } \frac{13N}{39}\right)$ is 8 since the bigger one exceeds the smaller one by 8. Hence, $\frac{2N}{39} = 8$

(5) Next, we will seek the value of N using the # line:



Since $\frac{2N}{39} = 8 \Rightarrow \frac{2}{39}$ of $N = 8$, then $\frac{1}{39}$ of $N = 4$.

Thus, we add all the $\frac{1N}{39}$ pieces of length 4 each all the way up to $\frac{39}{39}N = N$. We obtain a line of $39 \times 4 = 156$ units

(6) Hence, $\boxed{\text{N is 156 units}}$

good.