1. Let $V \subseteq \mathbb{A}^m$ and $W \subseteq \mathbb{A}^n$ be varieties and $\phi : V \to W$ a regular map. The subset $\Gamma_\phi$ of $V \times W \subseteq \mathbb{A}^{m+n}$ consisting of points of the form $\{(v, \phi(v)) : v \in V\}$ is called the graph of $\phi$. Show that $\Gamma_\phi$ is a variety isomorphic to $V$.

2. Suppose $V \subset \mathbb{A}^n$ is a variety with irreducible components $V_1, \ldots, V_r$. A regular function $\phi \in k[V]$ naturally gives rise to an $r$-tuple of regular functions $\phi_i \in k[V_i]$ which have the property that they agree on intersections: $\phi_i|_{V_i \cap V_j} = \phi_j|_{V_i \cap V_j}$. Do all such $r$-tuples arise in this way?

3. Show that the complex plane curves $y^2 = x^{2k+1}$ are irreducible and pairwise birationally equivalent, for $k = 0, 1, 2, \ldots$

4. CLO Problem 5-$§3$-12 on p. 238 (on dimension)

5. (This is Shafarevich I-I-$§3$ Example 2). Let $X = V(x^3 + y^3 + z^3 - 1) \subset \mathbb{A}^3$ over a field of characteristic $\neq 3$. Then $X$ contains lines $L_1 = V(x + y, z - 1)$ and $L_2 = V(x + \omega y, z - \omega)$ for $\omega \neq 1$ a cube root of 1. Let $E = V(z)$ be a plane.

Show that there is an open subset $U \subset X$ such that through every point $x \in U$, there is a unique line through $x$ that meets $L_1$ and $L_2$. Write down the rational map $f : X \dashrightarrow E$ that sends $x$ to the intersection of that line with $E$, and write down the map $g : E \dashrightarrow X$ such that $f \circ g = id_E$ and $g \circ f = id_X$. 
