1. Let $\Delta$ be a collection of subsets of $\{1, \ldots, n\}$ such that if $S \in \Delta$ and $T \subseteq S$ then $T \in \Delta$. For $i = 1, \ldots, n$, let $f_i$ denote the number of sets in $\Delta$ of size $i$. Let

$$I_{\Delta} = \langle \prod_{j \in S} x_j : S \subseteq \{1, \ldots, n\}, S \not\in \Delta \rangle \subset k[x_1, \ldots, x_n].$$

Compute the Hilbert polynomial of $k[x_1, \ldots, x_n]/I_{\Delta}$.

2. Compute the Hilbert polynomial of the image of a hypersurface of degree $D$ in $\mathbb{P}^n$ under the $d^{th}$ Veronese embedding.

3. Show that for any homogeneous ideal $I \subseteq k[x_1, \ldots, x_n]$, there is a monomial ideal $J$ whose Hilbert function is the same as the Hilbert function of $I$.


Three exercises on lines in $\mathbb{P}^3$.

5. E-H Exercise III-67

6. E-H Exercise III-68

7. E-H Exercise III-69