

Homework – Week 7

Sets, Maps and Knots

Math 101 – Harvard University – Spring 2000

Due Monday, 7 November 2005

Problems from Fraleigh: 13 (1,2,3,5,6); 16 (3).

1. Let $|A| = n$ and let $S(A)$ be the group of permutations of A . Let $\mathcal{P}_k(A) = \{B \subset A : |B| = k\}$. That is, $\mathcal{P}_k(A)$ consists of all the subsets of A of size k . A permutation $\sigma : A \rightarrow A$ acts on $\mathcal{P}_k(A)$ by

$$\sigma \cdot B = \sigma(B) = \{\sigma(b) : b \in B\}.$$

- (a) Is the action of $S(A)$ on $\mathcal{P}_k(A)$ transitive?
- (b) Let $H_B \subset S(A)$ be the subgroup of permutations σ such that $\sigma \cdot B = B$. Describe H_B and compute its order.
- (c) Express $|\mathcal{P}_k(A)|$ in terms of $|S(A)|$ and $|H_B|$, and write the final result in terms of n and k .
2. (a) Show how a regular tetrahedron T can be drawn inside a cube C so that the 4 vertices of T lie among the 8 vertices of C . (The edges of T must all be the same length.)
- (b) Let $G \cong S_4$ be the symmetry group of C . Describe the subgroup $H \subset G$ of symmetries of C that preserve T , and compute $|H|$.