Final

Sets, Groups and Knots

Math 101 – Harvard University

Due by 12pm, Wednesday, 11 January, in room 325 Science Center

Instructions. Write your answers neatly on separate paper, stapled together, with your name on the first page. All work should be your own. Refer only to your class notes and the material from Halmos, Fraleigh and Adams that was covered in the course. All problems carry equal weight.

1. Let $E_i \subset \mathcal{P}(\mathbb{Z})$ be the collection of all subsets $A \subset \mathbb{Z}$ such that $|A| = i$. Show there is a bijection between E_2 and E_3.

2. Prove there is a set $E \subset \mathbb{R}$ such that every real number x can be written *uniquely* as $x = e + q$ with $e \in E$ and $q \in \mathbb{Q}$.

3. Show that any two elements of order 5 in S_9 are conjugate (that is, $x = gyg^{-1}$ for some $g \in S_9$). Show this fails for S_{10}.

4. (a) Find all the elements of order two in D_n. (You can express elements of D_n as $r^i f^j$.) (b) Determine when 2 such elements are conjugate. (c) How many 'types' of elements of order two does D_n contain, if we regard conjugate elements as being of the same type?

5. Prove that any group G of order 34 contains an element of order 17.

6. Let L be the 3-component link 6^3_2 from Adams' table. (a) Compute a presentation for the fundamental group $G(L)$. (b) Find a surjective homomorphism $\phi : G(L) \to \mathbb{Z}^3 = \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

7. Continuing with the problem above: prove that $G(L)$ is nonabelian.

8. (a) Show that the knot 6_3 is equivalent to its mirror image. (b) Explain why no other knot of 6 or 7 crossings in Adam's table is equivalent to its mirror image.

9. Let K be the knot projection 5_1 in Adams' table. Compute $w(K)$, $\langle K \rangle$, $X(K)$ and $V(K)$.

10. Compute the Jones polynomial of the knot shown below.