Math 99r Tutorial topics

(2009-2010)

Fall 2009 Tutorial

Dynamics of analytic maps: small divisor problems

Description: The dynamical properties of some families of dynamical systems may depend on number-theoretical properties of real numbers which appear as parameters in the system. For example, the trajectory of a ball in a square billiard table will be either periodic or uniformly distributed accordingly to the rationality or irrationality of the angle at which one hits the ball. In general, numbers which are hard to approximate with rationals will produce more complicated dynamics. The tutorial will start from an introduction to dynamical systems and provide you with some examples of these phenomena, up to the result of Siegel-Brjuno-Yoccoz, which classifies the germs of analytic diffeomorphisms in one complex variable using continued fractions expansions.

Prerequisites: Math 113 (Complex Function Theory)

Contact: Giulio Tiozzo (tiozzo@math.harvard.edu)

Spring 2010 Tutorial

Combinatorial and geometric group theory

Description: We will try to understand some of the rich interplay between infinite groups and geometry and topology. We will especially explore the combinatorial properties of free groups and surface groups and how these properties relate to the topology of graphs and the topology and geometry of manifolds, especially that of hyperbolic space. A central theme will be that a group should be considered not only as an algebraic object but also as a geometric object, and that aspects of both the algebraic structure and geometric structure affect each other profoundly.

Prerequisites: Math 131 is strongly encouraged. Math 132 is desirable but not necessary. Though no hyperbolic geometry will be assumed, the content of Math 130 would provide useful background.

Contact: Thomas Koberda (koberda@math.harvard.edu)

Algebraic combinatorics: symmetric functions

Description: Algebraic combinatorics is one of the most rapidly developing fields of combinatorics. As the name suggests, it is the intersection of algebra and combinatorics, i.e. the application of algebra to solve enumerative problems and vice versa - the use combinatorial methods to understand algebraic structures like group representations, varieties, cohomology classes. The theory of symmetric functions is its main tool of establishing these connections. We will introduce the various classes of symmetric functions and study the relationships between them, which will then lead to important combinatorial and algebraic results. Examples of their applications will include permutation enumerations, partition identities, Young tableaux properties, the characters of the symmetric group, etc. In the meantime you will also get acquainted with a lot of combinatorial techniques and tricks.

Prerequisites: basic knowledge of linear algebra (e.g. Math 121 or 122) will be required; some familiarity with abstract algebra (Math 122 or Math 55a) would be helpful but not required.

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