Sample Questions from Past Qualifying Exams

This list may give the impression that the exams consist of a series of questions fired at the student one after another. In fact most exams have more the character of a conversation with considerable give and take. Hence this list cannot be expected to indicate accurately the difficulties involved.

The list indicates the professor associated to each question where available. Some have been in the MGSA files for a while, and this information has been lost (if it was ever there).

The listing by section is approximate, since some questions may fit under more than one heading.

Partial Differential Equations

- Let $\Omega \subset \mathbb{R}^n$ be an open, bounded, smooth domain. Let $f\colon \partial\Omega \to \mathbb{R}$ be continuous. Is there a solution to $\Delta u = 0$ in Ω, $u = f$ on $\partial\Omega$? If there is a solution, explain how it is found.
- Let $\{v_n\}$ be a sequence of harmonic functions in Ω. Assume $v_n \to v$ uniformly in norm and that $v_n(x) \nearrow v(x) \forall x \in \Omega$. Show that v is harmonic in Ω.
- Consider $u_{tt} = u_{xx} + u_{yy}$ with $u(x, y, 0) = 0$ in $x^2 + y^2 < 1$ and $u_y(x, y, 0) = 0$ everywhere. What is $u(0, 0, \frac{1}{2})$?
- Consider $(*)$ $u_t = u_{xx}$, $u(x, 0) = f(x)$. Do you know a solution? Suppose you don't know the fundamental solution of the heat equation, how would you derive a solution of $(*)$? What conditions would you impose on $f(x)$ for uniqueness?
- For one dimensional Laplace's, heat and wave equations give initial and/or boundary conditions that allow you to find solutions.
- State the mean value property for harmonic functions and explain how you prove it.
- Consider the inviscid Burgers' equation. Assume there is a curve across which the solution is discontinuous. State hte Rankine-Hugoniot condition. State the entropy condition analytically.
- Follow the steps below to give a heuristic derivation of the entropy condiiont (that a shoc will occur if $u_r < u_l$):
  (a) Consider $(*)$ $v_t + vv_x - \epsilon v_{xx} = 0$. Let $w = v_x$ and differentiate $(*)$ with respect to x. write the result in terms of w.
  (b) Use $w^2 \geq 0$ and the maximum principle for subsolutions of the heat equation to conclude that w is bounded for all x and $t > 0$.
  (c) Thus $v_x \leq M$. Explain how this implies that if $u_r > u_l$ you cannot get a shock.