Mathematics is a long conversation: A celebration of Barry Mazur
Conference Program

Monday, June 4

8:45 - 9:15 am Morning Refreshments

9:15-9:30 AM
Welcoming Remarks

9:30-10:30 AM Marie-France Vignéras, Institut de Mathématiques de Jussieu
Title: "Supersingularity"

Abstract: I will explore a relation between supersingular elliptic curves over a finite field k and supersingular representations of the group GL(2,F) when F is a local field of residue field k, and then a construction of supersingular representations for any reductive p-adic group G.

10:30 - 11:00 am Coffee Break

11:00-12:00 PM Preston Wake, University of California, Los Angeles
Title: "Eisenstein Ideal with Squarefree Level"

Abstract: In his influential paper "Modular curves and the Eisenstein ideal," Mazur studied congruences modulo p between cusp forms and the Eisenstein series of weight 2 and prime level N. In particular, he defined the Eisenstein ideal in the relevant Hecke algebra, and showed that it is locally principal. We'll discuss the analogous situation for certain squarefree levels N, and show that, while the Eisenstein ideal may not be locally principal, we can count the minimal number of generators and explain the arithmetic significance of this number. This is joint work with Carl Wang-Erickson.

12:00 - 2:00 pm Lunch

2:00-3:00 PM Matthew Emerton, University of Chicago
Title: "Globalization and Lifting of Local mod p Galois Representations"

Abstract: If K is a finite extension of $\mathbb{Q}_p$, and $\overline{r}: G_K \longrightarrow GL_n(\overline{\mathbb{F}}_p)$ is a continuous representation, here are two natural questions one can ask about $\overline{r}$. (1) Can $\overline{r}$ be globalized to a representation $\overline{p}: G_F \longrightarrow GL_n(\overline{\mathbb{F}}_p)$? Here F is some number field whose completion at some prime v above p equals K, and the statement that $\overline{p}$ is a globalization of $\overline{r}$ means that $\overline{p}$ restricted to the decomposition group at v recovers $\overline{r}$. In fact, to make the notion of globalization worthwhile, it's natural to require more of $\overline{p}$. For this talk, the natural property to require of $\overline{p}$ is that it be irreducible and automorphic (in a suitable sense). (2) Can one lift $\overline{r}$ to a p-adic representation r: $G_K \longrightarrow GL_n(\overline{\mathbb{Z}}_p)$?

Again, one might ask that such a lift have additional properties: e.g. be crystalline, perhaps with appropriately prescribed Hodge–Tate weights.

Since automorphic mod p representations are liftable (essentially by definition), an affirmative answer to (1) (in its stronger form) yields an affirmative answer to (2). Conversely, Toby Gee and I have shown (using techniques arising from the theory of potential automorphy of Galois representations) that if r admits a lift

that is crystalline and furthermore potentially diagonalizable, and if p is coprime to 2n, then (1) (in its stronger form) has an affirmative answer.

More recently, we have also shown that (2) has an affirmative answer in a strong sense: we are able to show that any $\overline{r}$ does admit a potentially diagonalizable crystalline lift (whose Hodge–Tate weights can furthermore be prescribed in an appropriate manner).

In the first part of this talk I will discuss questions (1) and (2) in detail, recalling previously known results and explaining some of the issues that arise when trying to answer them in general. In the second part of the talk, I will explain how Toby Gee and I are able to answer question (2) affirmatively. The key ingredient in our argument is the moduli stack of rank n etale (ϕ, Γ) - modules. This is a Noetherian formal algebraic stack whose special fibre is a variety over $\overline{\mathbb{F}}_p$ whose closed points are in natural bijection with the various $\overline{r}$ (of fixed dimension n), and whose versal ring at the point corresponding to $\overline{r}$ is the usual formal deformation ring for $\overline{r}$. I will explain some of the properties of this stack, and how we can use it to construct p-adic lifts. (As the preceding discussion makes clear, this is joint work with Toby Gee.)

3:00 - 3:30 pm Coffee Break

3:30–4:30 PM WILLIAM STEIN, UNIVERSITY OF WASHINGTON

Title: "Writing the book "Prime Numbers and the Riemann Hypothesis" with Barry Mazur"

Abstract: In 2004, Barry Mazur and I started writing the book "Prime Numbers and the Riemann Hypothesis," which we recently published with Cambridge University Press. I'll talk about what's in the book and why, describe some of the technical aspects of production of the book, and tradeoffs for us between self publishing online versus with a traditional publisher.

5:00 - 7:00 pm Reception, The Austine & Chilton McDonnell Common Room, Science Center, 4th Floor

TUESDAY, JUNE 5

8:45 - 9:00 am Morning Refreshments

9:00–10:00 AM HECTOR PASTEN, HARVARD UNIVERSITY

Title: "Modularity, Special Points, and the ABC Conjecture"

Abstract: I will explain some new connections between modular forms and the ABC conjecture, and I will outline how these ideas lead to new unconditional bounds for the latter. This approach focuses on the degree of modular parametrizations from Shimura curves to elliptic curves, and comparisons between the degrees of several of these maps. While the ABC conjecture still seems out of reach, the results proved by these methods lie beyond the scope of the classical estimates obtainable by linear forms in logarithms.

10:00 - 10:30 am Coffee Break

10:30-11:30 AM ROBERT POLLACK, BOSTON UNIVERSITY
Title: "Mu-invariants of Modular Forms in Hida Families"

Abstract: In this talk we study the variation of mu-invariants in Hida families. We show these invariants are unbounded and in some cases directly related to p-adic L-functions of Dirichlet characters. In the case of X_0(11) and p=5 where Mazur first noticed a positive mu-invariant, we give a simple and explicit formula for how this mu-invariant varies in the Hida family attached to this elliptic curve.

11:30 - 12:00 pm Coffee Break

12:00-1:00 PM FRANK CALEGARI, BOSTON UNIVERSITY
Title: "Potential Modularity of Abelian Surfaces"

Abstract: I will discuss joint work (in progress) with George Boxer, Toby Gee, and Vincent Pilloni, in which we prove that all abelian surfaces over totally real fields are potentially modular.

1:00 - 2:30 pm Lunch

2:30-3:30 PM JACOB LURIE, HARVARD UNIVERSITY
Title: "Full Level Structures on Elliptic Curves"

Abstract: Let p be a prime number, and let M denote the moduli space of elliptic curves equipped with an infinite level structure at the prime p. Scholze has shown that the generic fiber of M is a perfectoid space. In this talk, I'll discuss an integral refinement of this result, as well as some "approximate" results, which hold at finite level.

3:30 - 4:00 pm Coffee Break

4:00-5:00 PM JASON STARR, STATE UNIVERSITY OF NEW YORK, STONY BROOK
Title: "Rational Simple Connectedness and PAC Fields"

Abstract: Barry Mazur proposed the notion of "rational simple connectedness" as an algebro-geometric analogue of simple connectedness, leading to analogues of topological obstruction theory. I will focus on applications (the Period-Index Theorem, Serre's "Conjecture II," variants of Ax's conjecture that "PAC implies C1") for Ax's class of "PAC" (pseudo-algebraically closed) fields, and function fields / valuation fields whose residue field is PAC.

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Wednesday, June 6 — Interdisciplinary Program

9:15 - 9:30 am Morning Refreshments

9:30–10:00 am Nur Yalman, Senior Fellow, Harvard Society of Fellows
Announcements and Opening Remarks

10:00–11:30 am Literature/Poetry
Jane Hirshfield - Chancellor, Academy of American Poets
Jamaica Kincaid - Harvard University Department of African & African American Studies
Elaine Scarry - Harvard University Department of English

***Moderator*: Maria Tatar**, Folklore & Mythology and Germanic Languages & Literatures, Harvard University

11:30 - 1:00 pm Lunch

1:00 – 2:30 pm History of Science
Stephanie Dick - University of Pennsylvania
Peter Galison - Pellegrino University Professor, Harvard University Department of Physics
Alma Steingart - Harvard University Department of the History of Science

***Moderator*: Keith Hilles-Pilant**, Executive Director, Math for America/Boston Associate, Harvard University Department of the History of Science

2:30 - 2:50 pm Coffee Break

2:50–4:20 pm Philosophy/Law/Physics
Michel Chaouli - Indiana University
Noah Feldman - Harvard Law School
Andrew Strominger - Harvard University Department of Physics

***Moderator*: William Mills Todd III**, Harry Tuchman Levin Professor of Literature and Professor of Comparative Literature, Harvard University Department of Slavic Languages & Literatures

4:20 - 4:40 pm Coffee Break

4:40–5:40 pm Manjul Bhargava, Princeton University
Public Lecture: "Poetry, Drumming, and Mathematics"

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Thursday, June 7

8:45 - 9:00 am Morning Refreshments

9:00-10:00 am Cathy O'Neil, O'NEIL RISK CONSULTING & ALGORITHMIC AUDITING (ORCAA)

Title: "Big data, Inequality, and Democracy: What Can Mathematics Offer?"

Abstract: We live in the age of the algorithm. Increasingly, the decisions that affect our lives—where we go to school, whether we get a job or a car loan, how much we pay for health insurance, what news we see on social media—are being made not by humans, but by mathematical models. The models being used today are opaque, unregulated, and uncontestable, even when they're wrong.

What's worse is they're defended as fair and objective in the name of mathematics. What can a mathematician do to push back? Cathy will discuss the latest research that tries to address this urgent question.

10:00 - 10:30 am Coffee Break

10:30-11:30 am Haruzo Hida, University of California, Los Angeles

Title: "Galois Deformation Ring and its Base Change to a Real Quadratic Field"

Abstract: For almost all primes split in a real quadratic field, we describe how to determine the isomorphism class of the universal ordinary deformation ring of a 2-dimensional induced representation from the Galois group over the real field. As an application, we can write down explicitly its base change to the quadratic field and the adjoint Selmer group of the universal deformation.

11:30 - 12:00 pm Coffee Break

12:00-1:00 pm Glenn Stevens, Boston University

Title: "Modular Symbols, K-theory, and Eisenstein Cohomology"

Abstract: In this talk we will give an adelic construction of an object that we call the Kato-Beilinson modular symbol for $GL(2)$, extending constructions of Goncharov and Brunault. We obtain a modular symbol Ψ belonging to the compactly supported cohomology of arithmetic subgroups of $GL(2)$ and taking values in a group of distributions valued in K_2 of the tower of modular curves. We interpret Ψ as a "universal" L-value for modular forms and explain how it specializes to Kato's Euler systems, as well as its role in Fukaya and Kato's proof of Sharifi's conjecture. Our hope is that these ideas will also help us understand a conjecture of Darmon and Dasgupta about "elliptic units" associated to real quadratic fields.

1:00 - 2:30 pm Lunch

2:30–3:30 PM JORDAN ELLENBERG, UNIVERSITY OF WISCONSIN
Title: "Heights on Stacks"

Abstract: When I was Barry's student, one thing I learned from him was that Diophantine problems were often not problems about rational points on varieties, but about rational points on stacks. At the time we were talking about generalized Fermat equations, but the point of view is much more general; for instance, a number field with Galois group G can be thought of as a rational point on the classifying stack BG.

Two popular questions in number theory are:

1. How many degree-d number fields are there with discriminant at most X?
2. How many rational points are there on a cubic surface with height at most X?

Our expectations about the first question are governed by Malle's conjecture; about the second, by the Batyrev-Manin conjecture. The forms of the conjectures are very similar, predicting in both cases an asymptotic of the form $c\ X^a\ (\log X)^b$, and this is no coincidence: in light of the above description, both are questions about rational points on a stack, and both questions should be asking us to count rational points of height at most X.

A serious obstacle is that there is no definition of the height of a rational point on a stack. The main point of this talk is to propose a definition and try to convince you it's the right one. If there's time, I'll also argue that when we talk about heights with respect to a line bundle we have always secretly meant "vector bundle," or should have. (Joint work with Matt Satriano and David Zureick-Brown).

3:30 - 4:00 pm Coffee Break

4:00–5:00 PM AKSHAY VENKATESH, STANFORD UNIVERSITY
Title: "Derived Hecke Algebra for Weight One Forms and Stark Units"

Abstract: I will describe the derived Hecke algebra in the case of weight one forms and how, in this setting, it is conjecturally related to Stark units. I will then describe some joint work with Henri Darmon, Michael Harris, and Victor Rotger where we establish this relationship for dihedral forms. The proof involves recent ideas of E. Lecouturier about the Eisenstein ideal, and the derivative of the Riemann zeta function at -1 (in the sense of Mazur-Tate) plays an important role.

FRIDAY, JUNE 8

8:45 - 9:00 am Morning Refreshments

9:00–10:00 AM WEI ZHANG, MIT
Title: "Selmer Groups for Rankin-Selberg L-functions of GL(2) x GL(3)"

Abstract: Let Π (resp. Σ) be a cohomological (for the trivial coefficient) cuspidal automorphic representation of GL(3) (resp. GL(2)) over a CM number field, and assume that they are base change from unitary groups. We prove the following theorem: if the Rankin-Selberg L-function $L(\Pi \times \Sigma, s)$ does not vanish at its center, then the associated ℓ-adic Bloch-Kato Selmer group vanishes (for primes ℓ where the mod ℓ Galois

representations satisfy certain mild conditions). The conditions on ℓ come from Euler system type argument. We will discuss some examples from elliptic curves. This is a joint work with Yifeng Liu, Yichao Tian, Liang Xiao, and Xinwen Zhu.

10:00 - 10:30 am Coffee Break

10:30-11:30 AM ALEXANDRA SHLAPENTOKH, EAST CAROLINA UNIVERSITY

Title: "Defining Valuation Rings and Other Definability Problems in Number Theory"

Abstract: We discuss questions concerning first-order and existential definability over number fields and function fields in the language of rings and its extensions. In particular, we consider the problem of defining valuations rings over finite and infinite algebraic extensions of $\mathbb{Q}$, and over function fields of arbitrary characteristic.

11:30 - 12:00 pm Coffee Break

12:00-1:00 PM PERSI DIACONIS, STANFORD UNIVERSITY

Title: "Barry Mazur as an Applied Mathematician"

Abstract: Mathematicians are sometimes asked to help 'outsiders' with applied problems. I'll report three of Barry's deviations in this direction. The first is about shuffling cards (OK, OK and random walk on finite groups of Lie type). The second is about wandering around on the streets of Paris (OK and random walk with reinforcement and the sigma model). The third is about Tomography (OK there IS an elliptic curve).

1:00 - 2:00 pm Farewell Lunch, The Austine & Chilton McDonnell Common Room, Science Center, 4th Floor