Math 99r Tutorial topics

(2008-2009)

Fall 2008 Tutorials

Tropical Geometry

Description: In enumerative algebraic geometry, one counts the number of solutions to a system of polynomial equations using geometric techniques; in tropical geometry, one counts them by counting!— Using an exotic 'tropical' definition of plus and times, one turns polynomial functions into piecewise-linear functions and turns their solution sets from complex algebraic varieties into simple line graphs. Everything numerical about the varieties can be seen at a glance by looking at their graphs, and various famous formulas that take pages of careful algebra to prove for varieties are consequences of simple combinatorial facts like Euler's formula V - E + F = 1 for planar graphs.— Even a hard theorem like the Caporaso-Harris formula for Gromov-Witten invariants will be amenable to our elementary tropical methods.

Prerequisites: The contents of Math 137 (Algebraic Geometry) make a nice complement to this course and motivate its goals, but are not required.

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Algebraic Graph Theory

The rapidly expanding area of algebraic graph theory uses two different branches of algebra to explore various aspects of graph theory: group theory (for studying graph symmetry) and linear algebra (for spectral theory). We start with basic definitions of graphs and then start looking at isomorphisms and automorphisms of graphs. We will translate the notion of coloring into its algebraic equivalent, homomorphisms, and study some interesting examples. Besides, we will also study spectral graph theory (i.e. the study of the eigenvectors and eigenvalues of matrices associated with graphs), and applications.

Prerequisites: Math 122 (Algebra I) or equivalent.

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Spring 2009 Tutorial

Morse Theory

Description: In this course we will see a method of using "good functions" on a manifold to study its topological structure, known as the Morse theory nowadays. This theory has not only provided many new and more direct approaches to the topics in classical algebraic topology but also had enormous application to various branches of math, including Smale's proof of the generalized Poincare conjecture, calculus of variations, the study of moment maps in symplectic geometry, and in mathematical physics. It appears in almost all fields related to differentiable manifolds.

On the other hand, (projective) algebraic varieties are objects which are defined by polynomial equations and are algebraic in nature. However, in this course, we will see how to understand the geometric feature of these algebraic objects by viewing them as differentiable manifolds and apply the Morse theory to them.

Prerequisites: Math 131 (Topology) and abstract linear algebra.

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